Calculus II
Exam 2, Fall 2002, Answers

Find all the integrals. Remember that definite integrals should have numerical answers.

1a. $\int x\ln(2x)dx$

Answer. Integrate by parts so that the logarithm disappears: let $u=\ln(2x),\ du=dx/x$ (notice the cancellation of the 2's), $dv=xdx,\ v=x^2dx/2$:

$$\int x\ln(2x)dx=\frac{x^2}{2}\ln(2x)-\frac{1}{2}\int xdx=\frac{x^2}{2}\ln(2x)-\frac{x^2}{4}+C\ .$$

1b. $\int \frac{\ln(2x)}{x}dx$

Answer. As we saw above, letting $u=\ln(2x),\ du=dx/x$, we have

$$\int \frac{\ln(2x)}{x}dx=\int udu=\frac{u^2}{2}+C=\frac{\ln(2x)}{2}+C=\frac{\ln x}{2}+C\ .$$

---

2. $\int_2^4 \frac{dx}{x^2-1}$

Answer. We have the partial fractions expansion

$$\frac{1}{x^2-1}=\frac{1}{2}\left(\frac{1}{x-1}-\frac{1}{x+1}\right)\ ,$$

so

$$\int_2^4 \frac{dx}{x^2-1}=\frac{1}{2}(\ln(x-1)-\ln(x+1))\big|_2^4=\frac{1}{2}(\ln 3-\ln 5-(\ln 1-\ln 3))=\frac{1}{2}\ln\left(\frac{9}{5}\right)\ .$$

---

3. $\int \tan^2 xdx$

Answer. Alas, $\tan^2 x=\sec^2-1$, so

$$\int \tan^2 xdx=\int(\sec^2-1)dx=\tan x-x+C\ .$$

4a. $\int e^x(e^{2x}+1)dx$

Answer. $\int e^x(e^{2x}+1)dx=\int(e^{3x}+e^x)dx=\frac{1}{3}e^{3x}+e^x+C\ .$

4b $\int x(e^{2x}+1)dx$

Answer. Here we must use integration by parts: $u=x,\ du=dx,\ dv=(e^{2x}+1)dx,\ v=(1/2)e^{2x}+x$:

$$\int x(e^{2x}+1)dx=x\left(\frac{1}{2}e^{2x}+x\right)-\int\left(\frac{1}{2}e^{2x}+x\right)dx=\frac{x}{2}e^{2x}+x^2-\frac{1}{4}e^{2x}-\frac{x^2}{2}+C$$

$$= (\frac{x}{2} - \frac{1}{4})e^{2x} + \frac{x^2}{2} + C \ .$$

---

5. $\int_1^2 \frac{dx}{x^2(x+1)}$

Answer. We have a partial fractions expansion of the form

$$\frac{1}{x^2(x+1)} = \frac{A}{x+1} + \frac{B}{x} + \frac{C}{x^2} \ .$$

Putting the expression on the right over the common denominator, we have equality of the numerators:

$$1 = Ax^2 + Bx(x+1) + C(x+1) \ .$$

$$\text{at } \ x = -1 \ \text{ we get } \ 1 = A \ ,$$

$$\text{at } \ x = 0 \ \text{ we get } \ 1 = C \ ,$$

$$\text{coefficient of } \ x^2 : \ \ 0 = A + B \ , \ \text{ so that } \ B = -1 \ .$$

Thus

$$\int_1^2 \frac{dx}{x^2(x+1)} = \int_1^2 \frac{dx}{x+1} - \int_1^2 \frac{dx}{x} + \int_1^2 \frac{dx}{x^2}$$

$$= (\ln 3 - \ln 2) - (\ln 2 - \ln 1) - (\frac{1}{2} - 1) = \ln 3 - 2\ln 2 + \frac{1}{2} = \frac{1}{2} + \ln(\frac{3}{4}) \ .$$