

Calculus II, Mathematics 1220-90

Examination 1, January 29,31,2004

You may use graphing calculators. Each problem is worth 20 points. You MUST show your work. Just the correct answer is not sufficient for any points.

1. Solve for x :

a) $2^x = 16(4^x)$

Solution. Since $16 = 2^4$ and $4 = 2^2$, this becomes $2^x = 2^4((2^2)^x) = 2^{4+2x}$, so we have to solve $x = 4 + 2x$, giving $x = -4$.

b) $(e^x)^2 = e^x e^2$

Solution. Similarly, the equation is $e^{2x} = e^{x+2}$, so we solve $2x = x + 2$, or $x = 2$.

2. Differentiate:

a) $f(x) = x \ln(x^2)$

Solution. First write $f(x) = 2x \ln x$. Now differentiate:

$$f'(x) = 2 \ln x + 2x \frac{1}{x} = 2 \ln x + 2 .$$

b) $g(x) = x(e^{x^2})$

Solution : $g'(x) = e^{x^2} + x(e^{x^2})(2x) = (e^{x^2})(1 + 2x^2) .$

3. In 10 years one kilogram of a certain radioactive element decays to .987 kg. What is the half life of this element?

Solution. Use the equation $P = P_0 e^{rt}$, with $P_0 = 1$, $P = .987$, $t = 10$ to find r :

$$.987 = e^{10r} \quad \text{so} \quad r = \frac{\ln(.987)}{10} = -.001309 .$$

Now the half-life T solves $.5 = e^{-.001309T}$, or

$$T = -\frac{\ln(.5)}{.001309} = 52.97 .$$

4. Find the definite integral:

$$\int_0^2 x(e^{x^2})dx .$$

Solution. Let $u = e^{x^2}$, $du = 2x(e^{x^2})dx$, and find

$$\int_0^2 x(e^{x^2})dx = \frac{1}{2} \int_1^{e^4} du = \frac{e^4 - 1}{2} .$$

5. Solve the initial value problem $y' - y = 3$, $y(0) = 5$.

Solution. The equation has the particular solution $y_p = -3$. The homogeneous equation $y' - y = 0$ has the general solution $y = Ke^x$. Thus the general solution is $y = Ke^x - 3$. Substitute the initial conditions: $5 = Ke^0$, so $K = 5$, and our solution is $y = 5e^x - 3$.