

Mathematics 1220 Calculus II, Examination 1, Sep 11,13, 2003

You may use graphing calculators. Each problem is worth 20 points. You MUST show your work. Just the correct answer is not sufficient for any points.

1., Differentiate

a. $f(x) = \ln[(x + 1)^2]$

Solution. Using the laws of logarithms we have $f(x) = 2 \ln(x + 1)$, so

$$f'(x) = \frac{2}{x + 1} .$$

However, if we just differentiate using the chain rule, we get

$$f'(x) = \frac{1}{(x + 1)^2} (2(x + 1)) = \frac{2}{x + 1} .$$

b. $g(x) = xe^{5x+3}$

Solution. Use the product rule:

$$g'(x) = e^{5x+3} + 5xe^{5x+3} = e^{5x+3}(1 + 5x) .$$

2. Find the definite integral $\int_0^3 2^{x^2} x dx$

Solution. Let $u = x^2$, $du = 2x dx$ and remember that $2 = e^{\ln 2}$. Thus

$$\begin{aligned} \int_0^3 2^{x^2} x dx &= \frac{1}{2} \int_0^9 2^u du = \frac{1}{2} \int_0^9 e^{(\ln 2)u} du \\ &= \frac{1}{2} \left[\frac{e^{(\ln 2)u}}{\ln 2} \right]_0^9 = \frac{1}{2 \ln 2} (2^9 - 2^0) = \frac{511}{2 \ln 2} = 368.61 . \end{aligned}$$

3. Find the definite integral $\int_0^3 \frac{\ln(x + 1)}{x + 1} dx$

Solution. Let $u = \ln(x + 1)$, $du = dx/(x + 1)$:

$$\int_0^3 \frac{\ln(x + 1)}{x + 1} dx = \int_{\ln 1}^{\ln 4} u du = \frac{u^2}{2} \Big|_0^{\ln 4} = \frac{1}{2} (\ln 4)^2 = .96091 .$$

4. When a certain amount of liquid milk at 100°C is put in the freezer (held at 0°C) it takes 2 minutes to reach 90°C . How long will it take for the milk to reach 10°C ?

Solution. Let $C(t)$ be the temperature of the milk in t minutes. The law here (Newton's Law of Cooling) is

$$\frac{dC}{dt} = k(C - 0) = kC ,$$

and the solution is $C(t) = C_0 e^{kt}$. Here $C_0 = 100$ and $C(2) = 90$. Thus

$$90 = 100e^{2k} , \quad \text{so} \quad k = \frac{1}{2} \ln(.9) = -.0527 .$$

Now that we know the value of k , we can find the value of t for which $C(t) = 10$:

$$10 = 100e^{-.0527t} , \quad \text{so} \quad t = -\frac{\ln(.1)}{.0527} = 43.71 \text{ minutes} .$$

5. Solve the initial value problem $y' + 5y = 5x$, $y(0) = 0$.

Solution. We seek a particular solution $y_p = Ax + B$. The equation gives us $A + 5(Ax + B) = 5x$. Equating coefficients, $5A = 5$ and $A + 5B = 0$. Thus $A = 1$ and $B = -1/5$, and $y_p = x - 1/5$.

The homogeneous equation is $y' + 5y = 0$, which has the general solution $y_h = Ke^{-5x}$. Thus the solution we seek is of the form

$$y = Ke^{-5x} + x - \frac{1}{5} .$$

Putting in the initial condition, we get $0 = K - 1/5$, so $K = 1/5$, and our answer is

$$y = \frac{1}{5}(e^{-5x} - 1) + x = x - \frac{1}{5} + \frac{1}{5}e^{-5x} .$$