Calculus II
Exam 1, Fall 2002, Answers

1. Differentiate:

a) $f(x) = e^x \ln x$

Answer. Use the product rule:

$$f'(x) = e^x \ln x + \frac{e^x}{x} .$$

b) $g(x) = e^{2x^2+3x-1}$

Answer. Use the chain rule:

$$g'(x) = e^{2x^2+3x-1}(4x+3) .$$

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2. Integrate

a) $\int e^{\ln x+1} dx$

Answer. By the rules of exponentials, $e^{\ln x+1} = e^{\ln x} e = xe$. Thus

$$\int \ln(3e^x) dx = \int exdx = e\frac{x^2}{2} + C .$$

b) $\int_0^3 e^x(e^{2x}+1)dx$

Answer. $= \int_0^3 (e^{3x}+e^x)dx = (\frac{e^{3x}}{3}+e^x)\Big|_0^3 = \frac{e^9}{3}+e-\frac{4}{3}$

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3. I want to invest \$5000 in a growth fund so that in 5 years i will have \$8000. What interest rate, compounded continuously will produce that growth?

Answer. The data give us the equation $8 = 5e^{5r}$, where r is the rate desired. Thus

$$r = \frac{1}{5}\ln(\frac{8}{5}) = .094 \quad \text{or} \quad 9.4\%$$

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4. A certain radioactive element decays so that in 47 years it has decreased to 80% its original size. What is its half-life?

Answer. Again, the decay equation is $A(t) = A_0 e^{-rt}$, where r is the rate of decay, t is the time, $A(t)$ is the amount at time t, and A_0 is the amount at time $t = 0$. We are told that $.8 = (1)e^{-r(47)}$, and we are asked to find the T such that $.5 = e^{-rT}$. From the first equation we find

$$-47r = \ln(.8) , \quad \text{so that} \quad r = \frac{\ln(.8)}{-47} = 4.75 \times 10^{-3} .$$

Then the half-life is the solution to $.5 = e^{-4.75\times10^{-3}T}$, so that

$$T = \frac{\ln(2)}{4.75\times10^{-3}T} = 146 \text{ years.}$$

5. Solve the initial value problem $xy' + y = x, \; y(2) = 5$.

Answer. First solve the homogeneous equation $xy' + y = 0$, for which the variables separate: $dy/y = -dx/x$. This integrates to $\ln y = -\ln x + C = \ln(1/x) + C$, which in turn exponentiates to $y = K/x$. So, we try $y = u/x$‘$, y' = u'/x + \cdots$ in the original equation, getting

$$xu'/x = x \quad \text{or} \quad u' = x \, ,$$

which has the solution $u = x^2/2 + C$. Thus

$$y = \frac{u}{x} = \frac{x}{2} + \frac{C}{x} \, .$$

The initial condition gives $5 = 1 + C/2$, so $C = 8$, and the answer is

$$y = \frac{u}{x} = \frac{x}{2} + \frac{8}{x} \, .$$