1220-90 Exam 1

Summer 2013

Name KEY

Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Please circle your final answer. The last page contains some useful identities.

1. (30pts) Find the indicated derivatives. No need to simplify.

   (a) (5pts) $D_x(\cosh(e^x))$

   $= \sinh(e^x)e^x$

   6

   (b) (5pts) $D_x(\ln \sqrt[3]{x})$

   $= \frac{1}{x^{1/3}} \cdot \frac{1}{3}x^{-2/3} = \frac{1}{3x}$

   6

   (c) (5pts) $D_x(2^{x^2+x})$

   $= (\ln 2)(2^{x^2+x})(2x+1)$

   6

   (d) (5pts) $D_x(\tan^{-1}(\sqrt{x^2-1}))$

   $= \frac{1}{(\sqrt{x^2-1})^2+1}\left(\frac{1}{2}(x^2-1)^{-1/2}\right)(2x) = \frac{1}{x^2}\left(\frac{x}{\sqrt{x^2-1}}\right)$

   $= \frac{1}{x\sqrt{x^2-1}}$

   6

   (e) (5pts) $D_x((3x)^{(2x)})$

   $(3x)^{(2x)} = (e^{\ln 3x})^{2x} = e^{2x\ln 3x}$

   6

   So

   $D_x((3x)^{2x}) = e^{2x\ln(3x)}(2\ln 3x + 2)$

   $= (3x)^{(2x)}(2\ln 3x + 2)$

30

2. (5pts) Find a formula for $f^{-1}(x)$ if

$$f(x) = \sqrt[3]{2x+8}.$$

Note: $f^{-1}(x)$ is the inverse of $f(x)$. It is **not** equal to $\frac{1}{f(x)}$.

5

$x = (2y+8)^{1/3}$

$x^3 = 2y+8 \Rightarrow y = \dfrac{x^3-8}{2}$

3. (10pts) Evaluate the following:

10

(a) $\sin^{-1}(\frac{\sqrt{3}}{2}) =$ $\pi/3$

(b) $\tan^{-1}(-1) =$ $-\pi/4$

(c) $\cos^{-1}(\cos(\frac{\pi}{2})) =$ $\pi/2$

(d) $\sin^{-1}(\sin(\frac{3\pi}{4})) =$ $\pi/4$

(e) $\sin(\cos^{-1}(\frac{4}{5})) =$ $3/5$

4. (10pts) There are initially 10 bacteria in a petri dish. After one hour, there are 32 bacteria. Let $P(t)$ denote the population of the bacteria after t hours. Note that $P(0) = 10$ and $P(1) = 32$.

(a) (6pts) Find a formula for $P(t)$ assuming the population grows exponentially.

6

$P(t) = 10e^{kt}$

$32 = 10e^{k} \rightarrow e^{k} = 3.2 \Rightarrow k = \ln(3.2)$

$P(t) = 10e^{\ln(3.2)t}$

or $P(t) = 10(3.2)^{t}$

(b) (4pts) How long will it take before there are 100 bacteria in the petri dish? No need to simplify.

4

$100 = 10e^{\ln(3.2)t}$

$10 = e^{\ln(3.2)t} \Rightarrow \ln 10 = \ln(3.2)\,t$

$t = \dfrac{\ln(10)}{\ln(3.2)}$

25

5. (24pts) Evaluate the following indefinite integrals. Remember $+C$!!

(a) $\int \sin(2x)\cos(3x)\,dx$

$= \int \frac{1}{2}\left(\sin(-x) + \sin(5x)\right)dx =$

6

$\cancel{=} -\frac{1}{2}\int \sin x\,dx + \frac{1}{2}\int \sin(5x)\,dx$

$= \frac{1}{2}\cos x - \frac{1}{10}\cos(5x) + C$

(b) $\int x\cos(2x)\,dx$

$= \frac{1}{2}x\sin(2x) - \frac{1}{2}\int \sin(2x)\,dx$

$u = x \quad du = dx$

$dv = \cos(2x)dx \quad v = \frac{1}{2}\sin(2x)$

$= \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C$

6

(c) $\int x^2 \ln x\,dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\,dx$

$u = \ln x \quad du = \frac{1}{x}dx$

$dv = x^2 dx \quad v = \frac{1}{3}x^3$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

6

(d) $\int \tan^3 x\,dx$ $= \int \tan^2 x \tan x\,dx$

$= \int (\sec^2 x - 1)\tan x\,dx$

6 $= \int \sec x \sec x \tan x\,dx - \int \frac{\sin x}{\cos x}dx$

$= \frac{1}{2}\sec^2 x + \ln|\cos x| + C$

24

6. (11pts) Evaluate the indefinite integral

$$\int x\sqrt{4-x^2}\, dx$$

in **two different** ways:

(a) (3pts) First, using the substitution $u = 4 - x^2$. $\Rightarrow du = -2x\,dx$

3 $\displaystyle\int x\sqrt{4-x^2}\,dx = -\frac{1}{2}\int u^{1/2}\,du = \frac{-1}{2}\cdot\frac{2}{3}u^{3/2} + C$

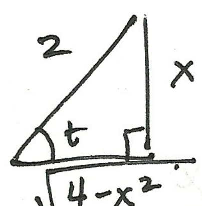
$$= \frac{-1}{3}(4-x^2)^{3/2} + C$$

(b) (8pts) Second, using the rationalizing substitution $x = 2\sin t$. $\Rightarrow dx = 2\cos t\,dt$
Note: You must show **all** of your steps to get full credit on this portion.

$$\int x\sqrt{4-x^2}\,dx = \int 2\sin t\cdot 2\cos t\cdot 2\cos t\,dt$$

8

$$= 8\int \cos^2 t\,\sin t\,dt$$



$u = \cos t$

$du = -\sin t\,dt$

$$= -8\int u^2\,du = \frac{-8}{3}u^3 + C$$

$\cos t = \dfrac{\sqrt{4-x^2}}{2}$

$$= -\frac{8}{3}\cos^3 t + C$$

$$= \frac{-8}{3}\left(\frac{\sqrt{4-x^2}}{2}\right)^3 + C$$

$$= \frac{-1}{3}(4-x^2)^{3/2} + C$$

7. (10pts) Find $\displaystyle\int \frac{1}{(x+3)(x-4)}\,dx$

$$\frac{1}{(x+3)(x-4)} = \frac{A}{x+3} + \frac{B}{x-4}$$

10

$$= \frac{A(x-4) + B(x+3)}{(x+3)(x-4)}$$

$$1 = A(x-4) + B(x+3) \Rightarrow B = \frac{1}{7},\ A = -\frac{1}{7}$$

$$\int \frac{1}{(x+3)(x-4)}\,dx = -\frac{1}{7}\int \frac{1}{x+3}\,dx + \frac{1}{7}\int\frac{1}{x-4}\,dx$$

$$= -\frac{1}{7}\ln|x+3| + \frac{1}{7}\ln|x-4| + C$$