MATH 1210-90 Fall 2011
Third Midterm Exam
INSTRUCTOR: H.-PING HUANG

LAST NAME ________________________________
FIRST NAME ______________________________
ID NO. ________________________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE SPECIFIED METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1 40 ______
PROBLEM 2 20 ______
PROBLEM 3 20 ______
PROBLEM 4 20 ______

TOTAL 100 ______
PROBLEM 1

(40 pt) Analyze the function.

\[ y = f(x) = \frac{x}{1 + x^2}. \]

(1) Domain and range.

(2) Symmetry.

(3) \(x\)- and \(y\)-intercepts.

(4) Find the first derivative of \( f \).

(5) Find the second derivative of \( f \).
(6) Find the critical points, if any.

(7) Find the inflection points, if any.

(8) Find the intervals where $f$ is increasing, and the intervals $f$ is decreasing.

(9) Find the intervals where $f$ is concave up, and the intervals $f$ is concave down.

(10) Find the asymptotes.

Sketch the graph of $f$. 
PROBLEM 2

(20 pt) Find the dimension of the right circular cylinder of greatest volume that can be inscribed in a given right circular cone.

**Hint:** Let $a$ be the altitude and $b$ be the radius of the base of the given cone. Find out the altitude, radius, and volume, respectively, of an inscribed cylinder.
PROBLEM 3

(20 pt) Use Newton’s method to find an approximation solution to the equation

\[ x^3 + x = -3 \]

as follows. Let \( x_1 = -1 \) be the initial approximation. What is the second approximation \( x_2 \)?
PROBLEM 4

(20 pt) Consider the differential equation:

\[ \frac{du}{dt} = -u^2(t^3 - t). \]

Find the particular solution of the above differential equation that satisfies the condition \( u = 4 \) at \( t = 0 \).