1210-90 Exam 2

Summer 2013

Name KEY

Instructions. Show all work and include appropriate explanations when necessary. A correct answer unaccompanied by work may not receive full credit. Please try to do all work in the space provided. Please circle your final answers.

1. (25pts) For this problem, consider the function

$$f(x) = 3x^5 - 5x^3 + 2.$$

(a) (4pts) Find $f'(x)$.

4

$$f'(x) = 15x^4 - 15x^2$$

(b) (4pts) Find the three critical points of $f(x)$.

4

$$0 = 15x^4 - 15x^2 = 15x^2(x^2-1) = 15x^2(x+1)(x-1)$$

cps: $-1, 0, 1$

(c) (4pts) Use the First Derivative Test to determine whether each critical point is a local minimum, a local maximum, or neither.

4

$+ \quad -1 \quad - \quad 0 \quad - \quad 1 \quad +$ sign f'

$x = -1 \Rightarrow$ local max

$x = 0 \Rightarrow$ neither

$x = 1 \Rightarrow$ local min

(d) (4pts) Find $f''(x)$.

4

$$f''(x) = 60x^3 - 30x$$

(e) (4pts) Find the inflection point(s) of $f(x)$, or show that there are no inflection points. It is acceptable to only list the x-values of the infection points.

4

$$0 = f''(x) = 60x^3 - 30x = 30x(2x^2-1) = 30x(\sqrt{2}x+1)(\sqrt{2}x-1)$$

$- \quad -1/\sqrt{2} \quad + \quad 0 \quad - \quad 1/\sqrt{2} \quad +$ sign f''

inf. pts: $x = -1/\sqrt{2}, 0, 1/\sqrt{2}$

(f) (5pts) Find the minimum and maximum **values** of $f(x)$ on the interval $[-1, 2]$.

5

Check endpts and cps:

$f(-1) = 4$

$f(0) = 2$

$f(1) = 0$

$f(2) = 58$

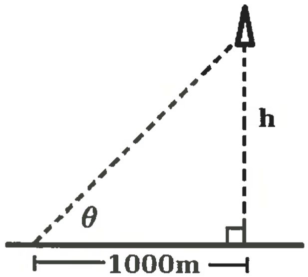
min = 0

max = 58

25

8

2. (8pts) A rocket launch is being watched by an observer 1000 meters away from the launch pad. When the rocket's height is 1000 meters, the observed angle of the rocket (labeled θ in the picture below) is increasing at a rate of $\frac{1}{20}$ radians per second. How fast is the rocket traveling at this time?



3 $\tan\theta = \frac{h}{1000} \Rightarrow h = 1000\tan\theta$

Diff both sides w.r.t t:

3 $\frac{dh}{dt} = 1000\sec^2\theta \, \frac{d\theta}{dt}$

When $h = 1000$, $\tan\theta = \frac{1000}{1000} = 1 \Rightarrow \theta = \frac{\pi}{4}$

So

$$\frac{dh}{dt} = 1000\sec^2(\pi/4)\left(\frac{1}{20}\right) = 1000(2)\left(\frac{1}{20}\right) = 100 \text{ m/s}$$

5

3. (5pts) Find the slope of the tangent line to the following curve at the point $(1, 2)$.

$$x^3 - 2xy^2 + y^3 = 1$$

Diff. both sides:

3 $3x^2 - 2y^2 - 4xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

Plug in $(x,y) = (1,2)$

$3 - 8 - 8\frac{dy}{dx} + 12\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = 5/4$

4

4. (a) (4pts) Find the equation of the tangent line to the graph of $y = \sqrt{x}$ at the point $(16, 4)$.

$f(x) = x^{1/2} \Rightarrow f(16) = 4$

$f'(x) = \frac{1}{2}x^{-1/2} \Rightarrow f'(16) = 1/8$

$y = 4 + \frac{1}{8}(x - 16)$

4

(b) (4pts) Use part (a) above to estimate $\sqrt{16.2}$.

$\sqrt{16.2} = f(16.2) \approx 4 + \frac{1}{8}(16.2 - 16) = 4 + \frac{1}{40}$

5

5. (5pts) Find the value c guaranteed by the Mean Value Theorem for $f(x) = x^3 - 2x$ on the interval $[0, 3]$

2 $f'(x) = 3x^2 - 2$

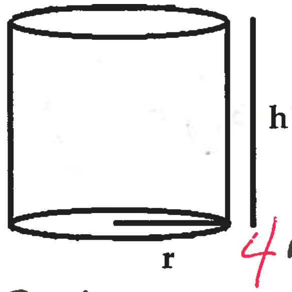
2 $3c^2 - 2 = f'(c) = \frac{f(3) - f(0)}{3 - 0} = \frac{21 - 0}{3} = 7.$

$3c^2 = 9$

$c^2 = 3 \Rightarrow c = \sqrt{3}$ /

26

6. (8pts) A closed cylindrical can is being manufactured to hold 250π cm^3 of liquid. What are the radius and height (labeled r and h in the picture below) of the can which can be made using the least amount of material? Minimize the surface area of the can $A = 2\pi r^2 + 2\pi r h$ subject to a fixed volume $V = \pi r^2 h$. **Note:** You must use calculus to get credit!!



8

$A = 2\pi r^2 + 2\pi r h$

$250\pi = \pi r^2 h \Rightarrow h = \frac{250}{r^2}$

So we want to minimize

4 $A(r) = 2\pi r^2 + 2\pi r\left(\frac{250}{r^2}\right) = 2\pi r^2 + \frac{500\pi}{r}$

Find cps:

$0 = A'(r) = 4\pi r - \frac{500\pi}{r^2} \Rightarrow \frac{500\pi}{r^2} = 4\pi r \Rightarrow r^3 = 125$

2 $\Rightarrow r = 5.$

Check $r=5$ is a local min:

-1 for missing

$A''(r) = 4\pi + 1000\pi r^{-3} \Rightarrow A''(5) > 0.$

2

When $r=5$, $h = \frac{250}{5^2} = 10.$ $r=5$, $h=10$

7. (16pts) Find the indicated antiderivatives:

-2 total missing $+C$

(a) (4pts) $\int (2x-3)\,dx$

4

$x^2 - 3x + C$

(b) (4pts) $\int \sqrt[5]{x}\,dx$ $= \int x^{1/5}\,dx$

4

$\frac{5}{6}x^{6/5} + C$

(c) (4pts) $\int (x^{-3} + \sin x)\,dx$

4

$-\frac{1}{2}x^{-2} - \cos x + C$

(d) (4pts) $\int (x^3+2x)^5(3x^2+2)\,dx$

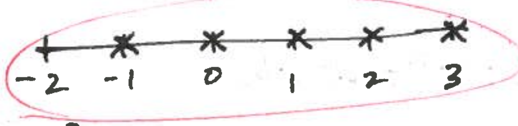
4

$\frac{1}{6}(x^3+2x)^6 + C$

24

5

8. (a) (5pts) Approximate $\int_{-2}^{3} |x|\, dx$ using a Riemann sum with 5 subintervals of equal length and the sample points being the right-endpoints of the subintervals.

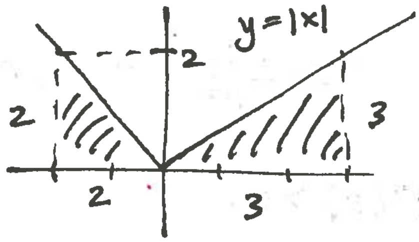


$\Delta x = \frac{3-(-2)}{5} = 1.$ 3

$$\int_{-2}^{3} |x|\, dx \approx 1\left(|-1| + |0| + |1| + |2| + |3|\right) = 7$$ 2

(b) (5pts) Find the exact value of $\int_{-2}^{3} |x|\, dx$. **Hint:** Use geometry and the area interpretation of the definite integral. What does the graph of $y = |x|$ look like?

5



$$\int_{-2}^{3} |x|\, dx = \text{shaded area}$$
$$= \tfrac{1}{2}(2)(2) + \tfrac{1}{2}(3)(3)$$
$$= 2 + \tfrac{9}{2} = \tfrac{13}{2}$$

9. (10pts) The pull of the Earth's gravity is slightly less on the top of a tall building than it is on the Earth's surface. Suppose we drop a ball from the top of the Sears Tower (1730 feet). We estimate that, as the ball drops, it experiences an acceleration of

$$a(t) = -31 - \frac{t}{10}$$

feet per second squared. The negative sign indicates that gravity acts downward.

(a) (5pts) Find $v(t)$, the velocity of the ball in feet per second after t seconds.

5

$v(t) = \int a(t)\, dt = \int \left(-31 - \frac{t}{10}\right) dt = -31t - \frac{t^2}{20} + C$

when $t=0$, $v(0)=0$ (ball just dropped) $\Rightarrow C=0.$

$v(t) = -31t - t^2/20$

(b) (5pts) Find an equation whose solution will give the time it takes for the ball to hit the ground. Do not attempt to solve.

5

Find when $s(t)=0$.

$s(t) = \int v(t)\, dt = -\frac{31}{2}t^2 - \frac{t^3}{60} + C$

when $t=0$, $s(0)=1730 \Rightarrow C=1730$

$0 = -31/2\, t^2 - t^3/60 + 1730$

10. (5pts) Compute x_2, the second approximation to the root of

$$x^3 - 2 = 0$$

using Newton's Method with initial approximation $x_1 = 1$.

5

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ 3

$f(x) = x^3 - 2 \Rightarrow f(1) = -1$

$f'(x) = 3x^2 \Rightarrow f'(1) = 3.$

$x_2 = 1 - \frac{-1}{3} = \frac{4}{3}$

25