1. (25pts) For this problem, consider the function

\[ f(x) = x^4 - 4x^3 + 4x^2 + 1. \]

(a) (4pts) Find \( f'(x) \).

(b) (5pts) Find the three critical points of \( f(x) \).

(c) (4pts) List the interval(s) on which \( f(x) \) is increasing and the interval(s) on which \( f(x) \) is decreasing.

(d) (4pts) Find \( f''(x) \).

(e) (4pts) Use the Second Derivative Test to determine whether each critical point found in part (b) is a local minimum or a local maximum.

(f) (4pts) Find the maximum and minimum values of \( f(x) \) on the interval \([−1, ∞)\). Write ‘DNE’ in the blank if there is none.

Maximum value = 

Minimum value =
2. (8pts) An ice cube is melting in the hot sun. Suppose the ice cube is losing volume at a rate of .24 cm³/min. How fast is the side length of the cube (labeled x in the picture below) decreasing when the volume of the ice cube is 8 cm³? Assume that the cube remains perfectly cubical at all times and recall that the volume of a cube is \( V = x^3 \).

3. (8pts) Find the equation of the tangent line to the following curve at the point \((1, -1)\).

\[ x^2 y + xy^2 + 2y = -2 \]

4. (5pts) Find the value \( c \) guaranteed by the Mean Value Theorem for \( f(x) = x^2 + x + 3 \) on the interval \([-1, 2]\)

5. (5pts) Compute \( x_2 \), the second approximation to the root of

\[ x^2 - 5x + 7 = 0 \]

using Newton’s Method with initial approximation \( x_1 = 3 \).
6. (10pts) A cylindrical can with an open top is being manufactured out of 48\(\pi\) cm\(^2\) of aluminum. What are the radius and height (labeled \(r\) and \(h\) in the picture below) of the can which holds the most volume? Maximize the volume of the can \(V = \pi r^2 h\) subject to the fixed surface area \(A = \pi r^2 + 2\pi rh\).

**Note:** You must use calculus to get credit!!

![Diagram of a cylindrical can](image)

7. (16pts) Find the indicated general antiderivatives: Remember +C!

   (a) (4pts) \(\int (x^3 + 2x + 1) \, dx\)

   (b) (4pts) \(\int (9 \sin x + 2) \, dx\)

   (c) (4pts) \(\int \frac{1}{x^2} \, dx\)

   (d) (4pts) \(\int (x^2 + 5)^3(2x) \, dx\)
8. (12pts) The graph of \( y = f(x) \) is found in Figure A below.

(a) (6pts) Approximate \( \int_{-5}^{5} f(x) \, dx \) using a Riemann sum with 5 subintervals of equal length and the sample points being the midpoints of the subintervals.

(b) (6pts) Find the exact value of \( \int_{-5}^{5} f(x) \, dx \). \textbf{Hint:} Use geometry and the area interpretation of the definite integral.

Figure A

9. (11pts) At time \( t = 0 \), a driver in a car traveling at 100 feet per second applies the brakes. Suppose the car decelerates at a constant 20 feet per second squared.

(a) (4pts) Find \( v(t) \), the velocity of the car in feet per second after \( t \) seconds.

(b) (3pts) How many seconds elapse before the car comes to a complete stop?

(c) (4pts) How many feet does the car travel before it comes to a complete stop?