Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Please circle your final answers.

1. (22pts) Let
   \[ f(x) = \frac{4x - 4}{x^2 - 4x + 3} \]
   Answers below may be values, DNE (does not exist), or ±∞. You must show your work.
   (a) (4pts) Compute \( \lim_{x \to 3^+} f(x) \)

   (b) (4pts) Compute \( \lim_{x \to 3^-} f(x) \)

   (c) (4pts) Compute \( \lim_{x \to 3} f(x) \)

   (d) (4pts) Compute \( \lim_{x \to +\infty} f(x) \)

   (e) (2pts) \( f(x) \) has a horizontal asymptote at \( y = \)______.

   (f) (2pts) \( f(x) \) has a vertical asymptote at \( x = \)______.

   (g) (2pts) \( f(x) \) is continuous everywhere except \( x = \)____________ (list all values).
2. (16 pts) Compute the following limits. Be sure to show your work.

(a) (4 pts) \( \lim_{x \to 0} \frac{x^2 - 2}{5 \cos x} \)

(b) (4 pts) \( \lim_{x \to 2} \frac{x^2 + x - 6}{x - 2} \)

(c) (4 pts) \( \lim_{x \to 0} \frac{\sin x \cos x}{x} \)

(d) (4 pts) \( \lim_{x \to 0} \frac{(3 + x)^2 - 9}{x} \)

3. (10 pts) Use the definition of the derivative to compute the derivative of \( f(x) = \sqrt{x} \); that is, compute the limit

\[
 f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

**Hint:** Use the algebraic identity \( \frac{\sqrt{a} - \sqrt{b}}{c} = \left( \frac{\sqrt{a} - \sqrt{b}}{c} \right) \left( \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}} \right) = \frac{a-b}{c(\sqrt{a} + \sqrt{b})} \).
4. (20pts) Compute the following derivatives using the derivative rules. There is no need to simplify.

(a) (5pts) $D_x (4x^5 + x^3 - 2x^2 + 9)$

(b) (5pts) $D_x (\sin x \cos x)$

(c) (5pts) $D_x (\frac{x^2}{1-x^3})$

(d) (5pts) $D_x (\cos (x^5 + x))$

5. (6pts) Suppose $f$ and $g$ are two functions whose values, and the values of their derivatives, are given by the following chart

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
<th>$g(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-4</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

For example, $f(0) = 1$, $g'(2) = 2$, etc. Use derivative rules to fill in the blanks:

If $F(x) = f(x)g(x)$, then $F'(1) =$

If $G(x) = \frac{f(x)}{g(x)}$, then $G'(1) =$

If $H(x) = f(g(x))$, then $H'(1) =$
6. (12pts) Consider the function \( f(x) = 2 + \frac{x}{1+x^2} \).

(a) (4pts) Find \( f'(x) \).

(b) (4pts) Find the equation of the tangent line to the graph of \( y = f(x) \) at \( x = 0 \).

(c) (4pts) At what points \( x \) is the tangent line to the graph of \( y = f(x) \) horizontal?

7. (9pts) An object moves along a horizontal coordinate line so that its position (in meters) at time \( t \) (measured in seconds) is given by \( s(t) = (t - 1)^3 - 12t + 6 \).

(a) (3pts) Find the velocity of the object at time \( t \).

(b) (3pts) Find the acceleration of the object at time \( t \).

(c) (3pts) At what time is the object the farthest to the left? In other words, when is the object's position the smallest (i.e. the most negative)?

8. (5pts) The graph of \( y = f(x) \) is given below. Use it to answer the following questions:

(a) ________ True (T) or False (F): \( f(x) \) is continuous on \((-4, 4)\).

(b) ________ True (T) or False (F): \( f'(1) > f'(3) \).

(c) \( f(x) \) is not differentiable at \( x = \) ________.