1210-90 Exam 1
Spring 2013

Name KEY

Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Please circle your final answers.

1. (20pts) Compute the following limits. Be sure to show your work. **Note:** Answers can be values, $+\infty$, $-\infty$, or DNE (does not exist)

   (a) $\lim_{x\to 0} \frac{1+x^2}{\cos x}$

   4

   Since $\lim_{x\to 0} \cos x = 1$,

   $$\lim_{x\to 0} \frac{1+x^2}{\cos x} = \frac{1}{1} = 1$$

   (b) $\lim_{x\to 2} \frac{x^2+x-6}{x-2}$

   4

   $$\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+3)}{x-2} = \lim_{x\to 2} x+3 = 5$$

   (c) $\lim_{x\to +\infty} \frac{5x^2-5}{3x^2+x}$

   4

   $$\lim_{x\to \infty} \frac{5x^2-5}{3x^2+x} = \lim_{x\to\infty} \frac{x^2(5-5/x^2)}{x^2(3+x/x^2)} = \lim_{x\to\infty} \frac{5-5/x^2}{3+1/x} = \frac{5}{3}$$

   (d) $\lim_{x\to 1} \frac{1}{x^2-1}$

   4

   $$\lim_{x\to 1^+} \frac{1}{x^2-1} = +\infty$$
   $$\lim_{x\to 1^-} \frac{1}{x^2-1} = -\infty$$
   $$\Rightarrow \lim_{x\to 1} \frac{1}{x^2-1} \text{ DNE}$$

   $\pm\infty$ -2

   (e) $\lim_{x\to 0} \frac{x^3-1}{x^2}$

   4

   when x is near 0, x^2 is positive and near zero, while x^3-1 is near -1. So denominator goes to zero and quotient is negative, so

   $$\lim_{x\to 0} \frac{x^3-1}{x^2} = -\infty$$

   DNE -2

2. (20 pts) Compute the following derivatives.

4pts - completely correct
~~4pts~~ - small mistake
~~4pts~~ 3pts
2pts - right idea
1 pts - something is correct

(a) $D_x(5x^2 + x - 2)$

4

$10x+1$

(b) $D_x((x^2+1)\sin x)$

4

$2x\sin x + (x^2+1)\cos x$

(c) $D_x(\frac{\cos x}{x})$

4

$\frac{-x\sin x - \cos x}{x^2}$

(d) $D_x(\sqrt{2x^2+8x})$ $= D_x((2x^2+8x)^{1/2})$

4

$= \frac{1}{2}(2x^2+8x)^{-1/2}(4x+8)$

$= \frac{2x+4}{\sqrt{2x^2+8x}}$

(e) $D_x(\left(\frac{x}{x+1}\right)^7)$

$= 7\left(\frac{x}{x+1}\right)^6 \cdot \left(\frac{x+1-x}{(x+1)^2}\right)$

4

$= \frac{7x^6}{(x+1)^8}$

3. (10pts) Compute the derivative of $f(x) = \frac{1}{x}$ by using **the definition of the derivative**; that is, compute the limit

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{h\,x(x+h)}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)}$$

$$= \frac{-1}{x^2}$$

+2 for correct answer not from limit

4. (10pts) Find the equation of the tangent line to the curve determined by the equation

$$x^2y + xy^3 - y = 8$$

at the point $(1, 2)$.

Differentiate w.r.t both sides

$$2xy + x^2\frac{dy}{dx} + y^3 + 3xy^2\frac{dy}{dx} - \frac{dy}{dx} = 0$$

Plug in $x=1, y=2$

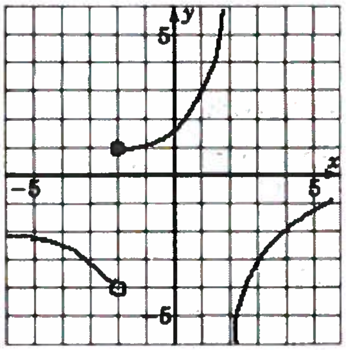
$$4 + \frac{dy}{dx} + 8 + 12\frac{dy}{dx} - \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -1.$$

Line with slope -1 passing through $(1,2)$

$$y - 2 = -1(x-1)$$

$$y = -x + 3$$

5. (10pts) Use the graph of the function $y = f(x)$ provided below to fill in the blanks.



2 (a) $\lim_{x\to -2^+} f(x) =$ 1.

2 (b) $\lim_{x\to -2^-} f(x) =$ -4.

2 (c) $\lim_{x\to -2} f(x) =$ DNE.

2 (d) The graph of $f(x)$ has a vertical asymptote at $x =$ 2.

2 (e) $\lim_{x\to -\infty} f(x) =$ -2.

6. (10pts) An object moves along the x-axis in such a way that its position at time at time t is given by

$$s(t) = -t^3 + 6t^2 - 9t + 1.$$

Assume that the units of the axis are measured in meters and t is measured in seconds.

(a) Find the velocity (in meters per second) of the object at time t.

4

$v(t) = s'(t) = -3t^2 + 12t - 9$

(b) On what time interval is the object moving to the right? Fill in the blanks: 1 $< t <$ 3

3

$v(t) = -3t^2 + 12t - 9 = -3(t^2 - 4t + 3) = -3(t-3)(t-1) > 0$

when $1 < t < 3$

(c) When is the acceleration of the object negative? Fill in the blank: $t >$ 2

3

$v'(t) = a(t) = -6t + 12 < 0$

2

when $t > 2$

7. (10pts) A snowball is being rolled to create the base of a snowman. Suppose the volume of the snowball is increasing at a rate of 10π cm^3/s. How fast is the radius of the snowball increasing when the snowball is 40 cm in diameter? **Hint:** Assume the snowball remains perfectly spherical. Recall that the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

10

$$V = \frac{4}{3}\pi r^3$$

$$dV/dt = 4\pi r^2 \, dr/dt$$

40 cm diameter $\Rightarrow r = 20$

$$\frac{dV}{dt} = 10\pi$$

$$10\pi = 4\pi(20)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{10\pi}{1600\pi} = \frac{1}{160} \text{ cm/s}$$

8. (10pts) Consider the function $f(x) = \sqrt[3]{x}$

(a) Find the linear approximation to the function $f(x) = \sqrt[3]{x}$ at $x = 64$. **Recall:** The linear approximation is the same as the equation for the tangent line.

7

$$f'(x) = \frac{1}{3}x^{-2/3} \Rightarrow f'(64) = \frac{1}{3}(64)^{-2/3} = \frac{1}{3}\cdot\frac{1}{16} = \frac{1}{48}$$ 2 1

when x is near 64,

$$f(x) \approx 4 + \frac{1}{48}(x-64)$$

3

$f(64)$ 1

(b) Use your answer above to estimate the value of $\sqrt[3]{65}$.

3

$$\sqrt[3]{65} = f(65) \approx 4 + \frac{1}{48}$$