1. (24pts) Compute the following limits. Be sure to show your work. Note: Answers can be values, +∞, −∞, or DNE (does not exist). An answer of DNE requires some explanation!

(a) \( \lim_{x \to 0} \sqrt{x^2 + 4} = \sqrt{0^2 + 4} = \sqrt{4} = 2 \)

(b) \( \lim_{x \to 3} \frac{x^3 - 7x + 12}{x - 3} = \lim_{x \to 3} \frac{(x-3)(x-4)}{(x-3)} = \lim_{x \to 3} x - 4 = -1 \)

(c) \( \lim_{x \to 0} \frac{\sin(2x)}{x} = \lim_{x \to 0} \frac{\sin(2x)}{2x} \cdot \frac{2x}{x} = (1)(2) = 2 \)

(d) \( \lim_{x \to 1} \frac{|x-1|}{x-1} \)

\[ \text{If } x > 1, \text{ then } |x-1| = x-1 \text{ and so } \lim_{x \to 1^+} \frac{|x-1|}{x-1} = \lim_{x \to 1^+} \frac{x-1}{x-1} = 1. \]

\[ \text{If } x < 1, \text{ then } |x-1| = 1-x \text{ and so } \lim_{x \to 1^-} \frac{|x-1|}{x-1} = \lim_{x \to 1^-} \frac{1-x}{x-1} = -1. \]

(e) \( \lim_{x \to +\infty} \frac{4x^2 + 9}{x^2 - x} = \lim_{x \to +\infty} \frac{4 + \frac{9}{x^2}}{1 - \frac{1}{x}} = \frac{4 + 0}{1} = 4 \)

(f) \( \lim_{x \to 0} \frac{x^3 - 4}{x^2} \)

When \( x \) is near 0, \( x^3 - 4 \) is near \( -4 < 0 \), while \( x^2 \) is near zero and positive. So \( \lim_{x \to 0} \frac{x^3 - 4}{x^2} = -\infty \)
2. (12pts) Suppose \( c \) is a constant and consider the piecewise-defined function

\[
f(x) = \begin{cases} 
  x^2 - 4x + c^2, & x < 1 \\
  -3cx + 1, & x \geq 1
\end{cases}
\]

(a) (4pts) Compute \( \lim_{x \to 1^-} f(x) \)

\[
\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 - 4x + c^2 = -3 + c^2
\]

(b) (4pts) Compute \( \lim_{x \to 1^+} f(x) \)

\[
\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} -3cx + 1 = -3c + 1.
\]

(c) (4pts) Find the value(s) of \( c \) that make \( f(x) \) continuous at \( x = 1 \).

\[
f(x) \text{ is continuous at } x = 1 \text{ if } \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)
\]

\[
-3 + c^2 = -3c + 1 \Rightarrow c^2 + 3c - 4 = 0 \Rightarrow (c+4)(c-1) = 0
\]

\( c = -4 \) or \( c = 1 \)

3. (10pts) Use the definition of the derivative to compute \( f'(1) \) for \( f(x) = x^3 + x \); that is, compute the limit

\[
f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}
\]

\[
= \lim_{h \to 0} \frac{(1+h)^3 + (1+h) - 2}{h}
\]

\[
= \lim_{h \to 0} \frac{1 + 3h + 3h^2 + h^3 + 1 + h - 2}{h}
\]

\[
= \lim_{h \to 0} \frac{4h + 3h^2 + h^3}{h}
\]

\[
= \lim_{h \to 0} (4 + 3h + h^2) = 4
\]
4. (20 pts) Compute the following derivatives. There is no need to simplify.

(a) \[ D_x(x^7 - 5x^5 + 3x) = 7x^6 - 25x^4 + 3 \]

(b) \[ D_x((x^3 + x)\sin x) = (3x^2 + 1)\sin x + (x^3 + x)\cos x \]

(c) \[ D_x(\frac{x^3}{x^2 + 1}) = \frac{(x^2 + 1)3x^2 - x^3(2x)}{(x^2 + 1)^2} = \frac{x^4 + 3x^2}{(x^2 + 1)^2} \]

(d) \[ D_x(\cos(7x^2 + 9)) = -\sin(7x^2 + 9)(14x) = -14x\sin(7x^2 + 9) \]

(e) \[ D_x\left(\left(\frac{\sin x}{x}\right)^5\right) = 5\left(\frac{\sin x}{x}\right)^4 \left(\frac{x \cos x - \sin x}{x^2}\right) \]

5. (6 pts) Find \( \frac{d^3y}{dx^3} \) if \( y = 5x^3 + x - \sin x \).

\[ \frac{dy}{dx} = 15x^2 + 1 - \cos x \]

\[ \frac{d^2y}{dx^2} = 30x + \sin x \]

\[ \frac{d^3y}{dx^3} = 30 + \cos x \]
6. (8 pts) Find the equation to the tangent line to the graph of the function \( f(x) = \frac{1}{(2-x)^2} \) at the point \((1,1)\).

\[ f(x) = \frac{1}{(2-x)^2} \]
\[ f(1) = \frac{1}{(2-1)^2} = \frac{1}{1} = 1. \]

\[ f'(x) = -2(2-x)^{-3} \]
\[ f'(1) = -2(2-1)^{-3} = -2(-1)^{-3} = \frac{2}{1} = 2. \]

Equation for tangent line at \( x = a \):
\[ y = f(a) + f'(a)(x-a) \]
So, equation for tangent line at \( x = 1 \):
\[ y = 1 + 2(x-1) = 2x - 1. \]

7. (8 pts) Let
\[ f(x) = \left( \frac{x}{1+x^2} \right)^7 \]

(a) (4 pts) Find the derivative \( f'(x) \).

\[ f'(x) = 7 \left( \frac{x}{1+x^2} \right)^6 \left[ \frac{(1+x^2)(1)-(x)(2x)}{(1+x^2)^2} \right] = 7 \frac{x^6(1-x^2)}{(1+x^2)^8} \]

(b) (4 pts) At what three points \( x \) is the tangent line to the graph of \( y = f(x) \) horizontal?

\[ f'(x) = 0 \Rightarrow \text{tangent line horizontal (slope zero)} \]
\[ 0 = f'(x) = 7 \frac{x^6(1-x^2)}{(1+x^2)^8} \Rightarrow 0 = x^6(1-x^2) \]
\[ = x^6(1+x)(1-x) \Rightarrow x = 0, \pm 1 \]

8. (12 pts) Examine the graph of the function \( f(x) \) below and fill in the blanks.

(a) \( \lim_{x \to -4} f(x) = \frac{3}{4} \)
(b) \( \lim_{x \to 4} f(x) = -2 \)
(c) List all values of \( x \), \(-6 < x < 5\), where \( f(x) \) is not continuous. \( x = -4, 2, 4 \)
(d) \( \text{ } \) True (T) or False (F): \( f'(1) > 0 \)
(e) \( \text{ } \) True (T) or False (F): \( f'(-2) > f'(-3) \)
(f) \( \text{ } \) True (T) or False (F): \( f \) is differentiable at \( x = 4 \).