MATH CIRCLE CONTEST
February 14, 2007

1. SECRET ADMIRERS?

B
E E
M M M
Y Y Y Y
V V V V V
A A A A A A
L L L L L L L
E E E E E E
N N N N N
T T T T
I I I
N N
E

In the above diagram, by beginning at the top "B" and moving down, one row at a time, to adjacent letters, how many ways can one spell BEMYVALENTINE? (For instance, one possibility would be to begin at the "B" move down and left for 6 rows to spell "BEMYVAL" and then move down and right for 6 more rows to finally spell "BEMYVALENTINE".)

2. FAIR TOSS

Suppose you play a game wherein you roll five dice, and score the total of the highest three. What is the probability that you score 16 or more?

3. ON AND ON AND ON AND ON...

Determine the value of the following expression:

$$\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\cdots}}}}.$$

4.

If a, b, and c are the roots of $x^3 - 5x^2 + 3x - 2$, find

$$\frac{a}{b} + \frac{a}{c} + \frac{b}{a} + \frac{b}{c} + \frac{c}{a} + \frac{c}{b}.$$

5. A FAMILIAR CHALLENGE

Assume

$$(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \cdots + a_{2n} x^{2n}$$

is an identity in x. Find

$$a_0 + a_2 + a_4 + \cdots + a_{2n}$$

in terms of n.