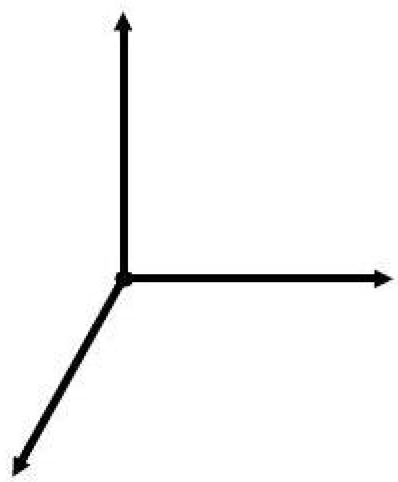
Math 2210 #7
Lines and Tangent Lines in 3-Space

A 3-D curve can be given parametrically by $x = f(t), y = g(t)$ and $z = h(t)$ where t is on some interval I and f, g, and h are all continuous on I. We could specify the curve by the position vector

$$\vec{r}(t) = f(t)\hat{\imath} + g(t)\hat{\jmath} + h(t)\hat{k}.$$



Given a point P_0, determined by the vector, $\vec{r}_0$ and a vector $\vec{v} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$, the equation $\vec{r} = \vec{r}_0 + \vec{v}t$ determines a line passing through P_0 at $t = 0$ and heading in the direction determined by $\vec{v}$.
(A special case is when you are given two points on the line, P_0 and P_1, in which case $\vec{v} = \overrightarrow{P_0P_1}$.)

$$\vec{r} = \langle x, y, z\rangle, \vec{r}_0 = \langle x_0, y_0, z_0\rangle \Rightarrow \langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + \langle a, b, c\rangle t$$
$$x = x_0 + at, y = y_0 + bt, z = z_0 + ct$$

These become the <u>parametric equations</u> of a line in 3D where a, b, c are called <u>direction numbers</u> for the line (as are any multiples of a, b, c).

EX 1

Find parametric equations of a line through $(2, -1, -5)$ and $(7, -2, 3)$.

Symmetric Equations for a line

$$x = x_0 + at, y = y_0 + bt, z = z_0 + ct \qquad \begin{array}{l} a \neq 0 \\ b \neq 0 \\ c \neq 0 \end{array}$$

This is the line of intersection between the two planes given by

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} \text{ and } \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

EX 2

Write the symmetric equations for the line through $(-2,2,-2)$ and parallel to $\langle 7,-6,3\rangle$.

EX 3

Find the symmetric equations of the line through $(\,-5,7,-2\,)$ and perpendicular to both $\langle 3,1,-3\rangle$ and $\langle 5,4,-1\rangle$.

EX 4

Find the symmetric equations of the line of intersection between the planes $x+y-z=2$ and $3x-2y+z=3$.

Tangent Line to a Curve

If $\vec{r} = \vec{r}(t) = f(t)\hat{\imath} + g(t)\hat{\jmath} + h(t)\hat{k}$ is a position vector along a curve in 3D, then

$$\vec{r}'(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h} \Rightarrow \vec{r}'(t) = f'(t)\hat{\imath} + g'(t)\hat{\jmath} + h'(t)\hat{k}$$

is a vector in the direction of the tangent line to the 3D curve. (This holds in 2D as well.)

EX 5

Find the parametric equations of the tangent line to the curve $x = 2t^2, y = 4t, z = t^3$ at $t = 1$.