

Math 2210 #5

The Cross Product

The Cross Product

$$\vec{u} \times \vec{v} = \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle$$

where $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$.

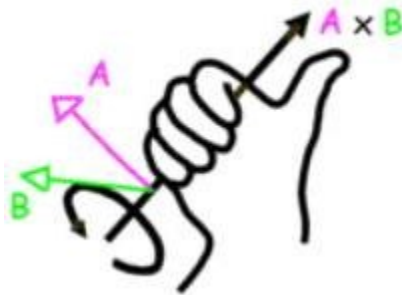
EX 1

If $\vec{a} = \langle 3, 3, 1 \rangle$ and $\vec{b} = \langle -2, -1, 0 \rangle$ and $\vec{c} = \langle -2, -3, -1 \rangle$, find $\vec{a} \times (\vec{b} \times \vec{c})$.

Theorem A

Let \vec{u} and \vec{v} be 3-D vectors and θ is the angle between them.
Then

1. $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0 = \vec{v} \cdot (\vec{u} \times \vec{v})$
2. \vec{u} , \vec{v} and $\vec{u} \times \vec{v}$ form a right-handed triple.
3. $\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \|\vec{v}\| \sin \theta$



Theorem B

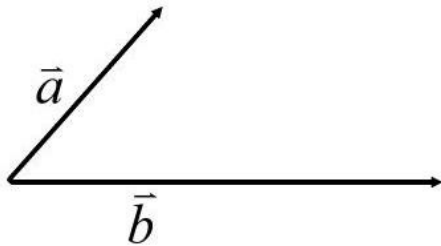
Two 3-D vectors, \vec{u} and \vec{v} are parallel iff $\vec{u} \times \vec{v} = 0$.

EX 2

Find the plane through these points.
 $P_1(-1,3,0)$, $P_2(5,1,2)$ and $P_3(4, -3, -1)$

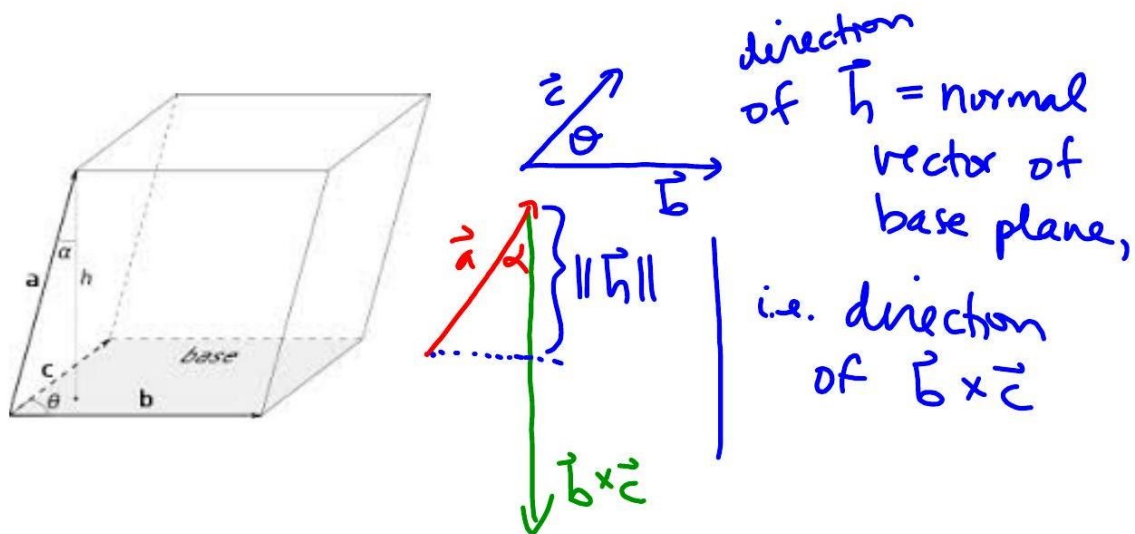
EX 3

Find the area of a parallelogram with vectors \vec{a} and \vec{b} as adjacent sides.



EX 4

Find the volume of a parallelogram prism(box) determined by the sides \vec{a} , \vec{b} and \vec{c} .



Theorem C Properties of Cross Product

\vec{u} , \vec{v} and \vec{w} are 3-D vectors and $k \in \mathfrak{R}$:

1. $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$
2. $\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w})$
3. $k(\vec{u} \times \vec{v}) = (k\vec{u}) \times \vec{v} = \vec{u} \times (k\vec{v})$
4. $\vec{u} \times \vec{0} = \vec{0} \times \vec{u} = \vec{0}$ and $\vec{u} \times \vec{u} = \vec{0}$
5. $(\vec{u} \times \vec{v}) \cdot \vec{w} = \vec{u} \cdot (\vec{v} \times \vec{w})$
6. $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$

$$\hat{i} \times \hat{j} = \hat{k} \quad \hat{j} \times \hat{k} = \hat{i} \quad \hat{k} \times \hat{i} = \hat{j}$$

EX 5

State whether each of the following expressions make sense or not. If it makes sense, tell if the result is a scalar or a vector.

5a)

$$\vec{u} \cdot (\vec{v} \times \vec{w})$$

5b)

$$\vec{u} + (\vec{v} \times \vec{w})$$

5c)

$$(\vec{a} \cdot \vec{b}) \times \vec{c}$$

5d)

$$(\vec{a} \times \vec{b} + k)$$

5e)

$$(\vec{a} + \vec{b}) \times (\vec{c} + \vec{d})$$

5f)

$$(\vec{a} \cdot \vec{b} + k)$$

EX 6

Find the equation of a plane through $(5, -1, 2)$ that is perpendicular to the line of intersection of the planes $4x - 3y + 2z = 1$ and $2x - y + z = 11$.