Math 2210 #3
A Geometric and Algebraic Approach to Vectors

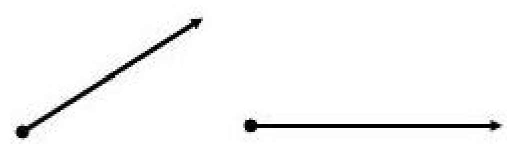
VECTORS (Geometric Approach)

<u>Scalar</u>

<u>Vector</u>

<u>Magnitude</u>

<u>Direction</u>



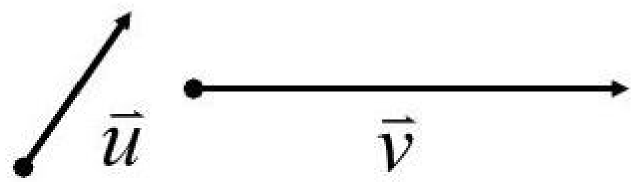
$\vec{u} = \vec{v}$ if they have the same magnitude and direction.

<u>zero vector</u> $\Rightarrow \vec{0}$ and $\vec{0} + \vec{u} = \vec{u} + \vec{0} = \vec{u}$

$-\vec{u} \Rightarrow$

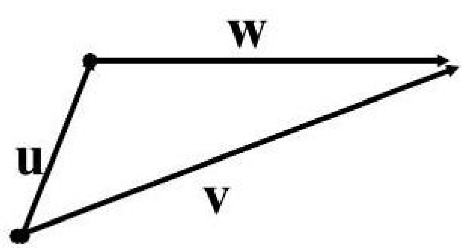
<u>scalar multiple</u> of $\vec{u} \Rightarrow c\vec{u}$, where c is a real number, means we have a vector in the direction of $\vec{u}$ but scaled in length.

Adding vectors $\Rightarrow \vec{u} + \vec{v}$



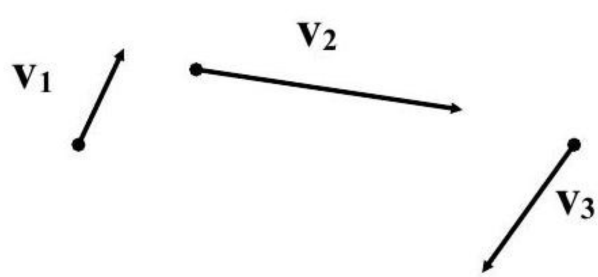
EX 1

Express **w** in terms of **u** and **v**.



EX 2

Draw **w** where $\mathbf{w} = \mathbf{v_1} + \mathbf{v_2} + \mathbf{v_3}$



EX 3

Mark pushes on a post in the direction S30°E with a force of 60 lbs. Dan pushes on the same post in the direction S60°W with a force of 80 lbs. What are the magnitude and direction of the resulting force?

EX 4

A ship is sailing due south at 20 mph. A man walks west across the deck at 3 mph. What are the magnitude and direction of his velocity relative to the surface of the water?

Vectors (Algebraic approach)

If we place our vector on a Cartesian Coordinate system with its tail at the origin, then its head will end at some point (u_l, u_2, u_3). We say that $\mathbf{u} = \langle u_l, u_2, u_3 \rangle u_1, u_2$ and u_3 are called components of $\boldsymbol{u}$.

$\mathbf{u} = \mathbf{v}$ iff $u_1 = v_1, u_2 = v_2$, and $u_3 = v_3$
$\mathbf{u} + \mathbf{v} = \langle u_l, u_2, u_3 \rangle + \langle v_l, v_2, v_3 \rangle = \langle u_1 + v_l, u_2 + v_2, u_3 + v_3 \rangle$
$-\mathbf{u} = \langle -u_l, -u_2, -u_3 \rangle$ $c\mathbf{u} = \langle cu_l, cu_2, cu_3 \rangle$
$\mathbf{0} = 0\mathbf{u} = \langle 0,0,0 \rangle$

Theorem A

For all vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ and the real numbers a and b

$$u + v = v + u$$

$$(u + v) + w = u + (v + w)$$

$$u + 0 = 0 + u$$

$$u + -u = 0$$

$$a(bu) = (ab)u$$

$$a(u + v) = au + av$$

$$(a + b)u = au + bu$$

$$1u = u$$

$$\|\mathbf{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$
$$\|c\mathbf{u}\| = |c|\|\mathbf{u}\|$$

EX 5

Let $\mathbf{u} = \langle -1,5,2 \rangle$, find $\|\mathbf{u}\|$ and $\|-3\mathbf{u}\|$.
Also, find a vector, $\hat{\mathbf{u}}$ with the same direction as $\mathbf{u}$ but with magnitude $= 1$. (This is called a unit vector)