Math 2210 #32
Stokes's Theorem

Remember this form of Green's Theorem:

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_R \nabla_{\mathbf{x}} \vec{F} \cdot \hat{k} dA$$

where

$$\vec{F}(x, y) = M(x, y)\hat{\imath} + N(x, y)\hat{\jmath},$$

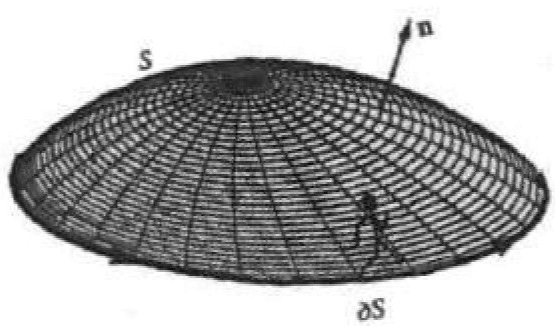
C is a simple closed positively-oriented curve that encloses a closed region, R, in the xy-plane.

It measures circulation along the boundary curve, C.

Stokes's Theorem generalizes this theorem to more interesting surfaces.

Stokes's Theorem

For $\vec{F}(x, y, z) = M(x, y, z)\hat{\imath} + N(x, y, z)\hat{\jmath} + P(x, y, z)\hat{k}$,

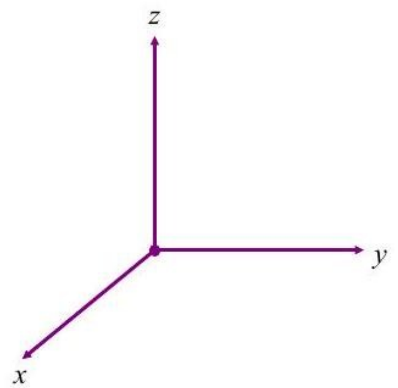


M, N, P have continuous first-order partial derivatives.
S is a 2 -sided surface with continuously varying unit normal, $\hat{n}$,
C is a piece-wise smooth, simple closed curve, positively-oriented that is the boundary of S,
T is the unit tangent vector to C,

then $\oint_C \vec{F} \cdot \hat{T} ds = \iint_S (\nabla \times \vec{F}) \cdot \hat{n} dS$

EX 1

Verify Stokes's Theorem for $\vec{F} = y^2\hat{\imath} - x\hat{\jmath} + 5z\hat{k}$ if S is the paraboloid $z = x^2 + y^2$ with the circle $x^2 + y^2 = 1$ as its boundary.



EX 2

Use Stokes's Theorem to calculate $\iint_S (\nabla_x \vec{F}) \cdot \hat{n} dS$ for $\vec{F} = xz^2\hat{\imath} + x^3\hat{\jmath} + \cos(xz)\hat{k}$ where S is the part of the ellipsoid $x^2 + y^2 + 3z^2 = 1$ below the xy-plane and $\hat{n}$ is the lower normal.

EX 3

Let S be a solid sphere. Show that $\iint_S \left(\nabla \times \vec{F}\right) \cdot \hat{n} dS = 0$

3a)

by using Stokes's Theorem

3b)

by using Gauss's Theorem