Math 2210 #30
Surface Integrals

Surface Integrals

Let G be defined as some surface, $z = f(x, y)$.
The surface integral is defined as

$$\iint_G g(x, y, z)dS, \text{ where } dS \text{ is a "little bit of surface area."}$$

To evaluate we need this <u>Theorem</u>:

Let G be a surface given by $z = f(x, y)$ where (x, y) is in R, a bounded, closed region in the xy-plane.

If f has continuous first-order partial derivatives and $g(x, y, z) = g(x, y, f(x, y))$ is continuous on R, then

$$\iint_G g(x, y, z)dS = \iint_R g(x, y, f(x, y))\sqrt{f_x^2 + f_y^2 + 1}dydx$$

EX 1

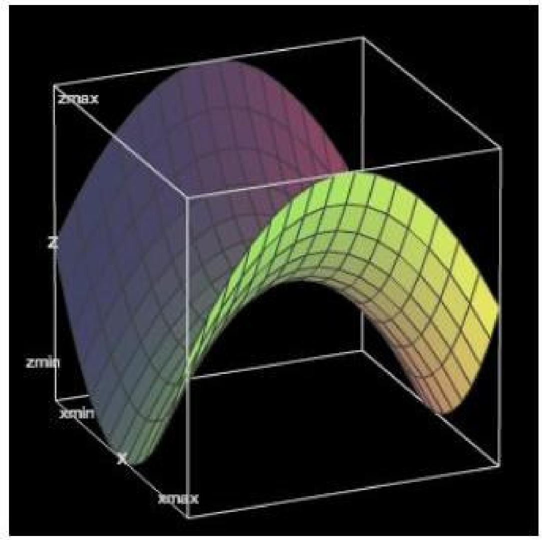
Evaluate $\iint_G g(x, y, z)dS$ given by $g(x, y, z) = x$, and G is the plane

$$\mathrm{x} + \mathrm{y} + 2\mathrm{z} = 4, x \in [0,1], y \in [0,1].$$

EX 2

Evaluate $\iint_G (2y^2 + z)dS$ where G is the surface

$$z = x^2 - y^2, \text{ with } R \text{ given by } 0 \leq x^2 + y^2 \leq 1.$$



EX 3

Evaluate $\iint_G g(x, y, z)dS$ where $g(x, y, z) = z$ and G is the tetrahedron bounded by the coordinate planes and the plane $4x + 8y + 2z = 16$.

Theorem

Let G be a smooth, two-sided surface given by $z = f(x, y)$, where (x, y) is in R and let $\vec{n}$ denote the upward unit normal on G. If f has continuous first-order partial derivatives and $\vec{F} = M\hat{\imath} + N\hat{\jmath} + P\hat{k}$ is a continuous vector field, then the flux of $\vec{F}$ across G is given by

$$\text{flux } \vec{F} = \iint_G \vec{F} \cdot \vec{n} dS = \iint_R \left[-Mf_x - Nf_y + P\right] dxdy$$

EX 4

Evaluate the flux of $\vec{F}$ across G where
$\vec{F}(x, y, z) = (9 - x^2)\hat{\jmath}$ and G is the part of the plane
$2x + 3y + 6z = 6$ in the first octant.