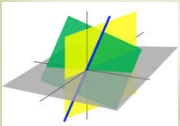
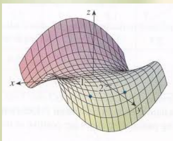
Cartesian Coordinates
in
3-Space

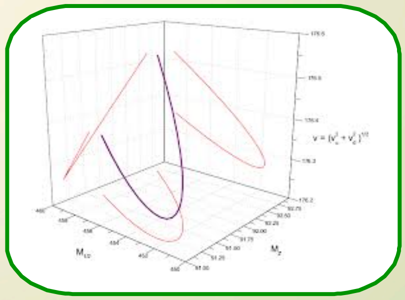


$$f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

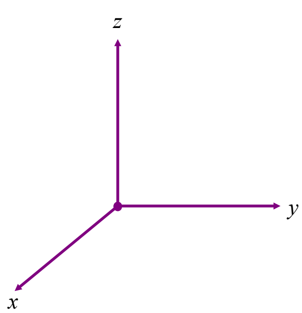
$$f_y = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

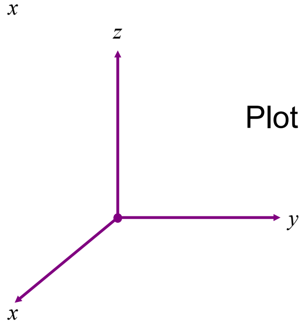


$$\int_0^1 \int_0^{2y} xy\,dx\,dy = \int_0^1 \left[\frac{x^2}{2}y\right]_{x=0}^{x=2y} dy$$
$$= \int_0^1 \frac{(2y)^2}{2} y\,dy = \int_0^1 2y^3\,dy$$
$$= \left[\frac{y^4}{2}\right]_{y=0}^{y=1} = \frac{1}{2}$$

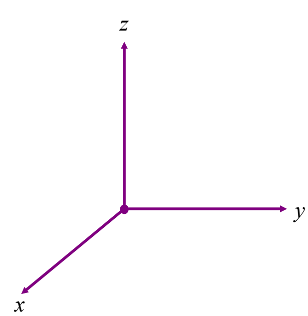


A point in 3-space is given by an ordered triple (x,y,z).



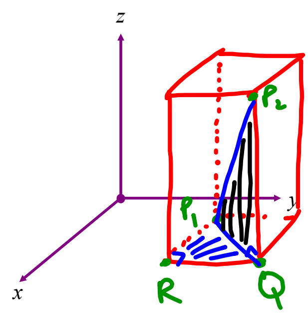


Plot ($2,3,4$)



Plot ($-1,4,-3$)

Distance Formula $d^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$



ΔP_1P_2Q and ΔP_1QR are both right triangles.

$\Rightarrow$ can use Pythagorean Thm

① $|P_1Q|^2 + |QP_2|^2 = |P_1P_2|^2$

and ② $|P_1R|^2 + |RQ|^2 = |P_1Q|^2$

$\Rightarrow$ sub ② into ①

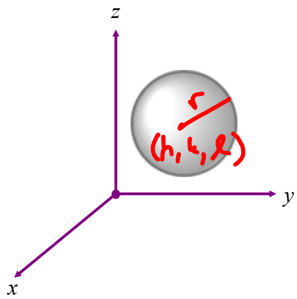
note: P_1Q means length of line segment from P_1 to Q

EX 1 Show that these points are vertices of an equilateral triangle.

$(4,5,3)$, $(1,7,4)$, $(2,4,6)$

Spheres All points (x,y,z) on a sphere are a fixed distance, r from the center.

$$r = \sqrt{(x-h)^2 + (y-k)^2 + (z-l)^2}$$



So the equation of a sphere with radius r and center (h,k,l) is $r^2 = (x-h)^2 + (y-k)^2 + (z-l)^2$

Midpoint of the segment (x_1,y_1,z_1) and (x_2,y_2,z_2)

$$m = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2} \right)$$

Ex 2

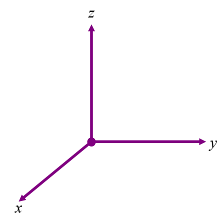
a) Find the center and radius of this sphere.

$$x^2 + y^2 + z^2 + 2x - 6y - 10z + 34 = 0$$

b) Find the equation of the sphere that has a diameter from $(-4,2,1)$ to $(8,3,6)$.

<u>Linear equations</u> in 3-space

$$Ax + By + Cz = D$$



EX 3 Graph $3x - 4y + 2z = 24$.

EX 4 Graph $3x + 4y = 12$.