Math 2210 #24
Triple Integrals in Cylindrical and Spherical Coordinates

Triple Integrals (Cylindrical and Spherical Coordinates)

$$\iiint_S f(x,y,z)dV = \int_{\theta_1}^{\theta_{21}} \int_{r_1(\theta)}^{r_2(\theta)} \int_{g_1(r,\theta)}^{g_2(r,\theta)} f(r\cos\theta\,, r\sin\theta\,, z) r dz dr d\theta$$

Note: Remember that in polar coordinates $dA = rdrd\theta$.

EX 1

Find the volume of the solid bounded above by the sphere $x^2 + y^2 + z^2 = 9$, below by the plane $z = 0$ and laterally by the cylinder $x^2 + y^2 = 4$. (Use cylindrical coordinates.)

EX 2

Find $\iiint_S f(x,y,z)dV$ for $f(x,y,z) = z^2\sqrt{x^2+y^2}$ and

$$\mathrm{S} = \{(x,y,z) \mid x^2 + y^2 \leq 4, -1 \leq z \leq 3\}$$

Spherical Coordinates

$$\iiint_S f(x,y,z)dV = \int_{\phi_1}^{\phi_2}\int_{g_1(\phi)}^{g_2(\phi)}\int_{\psi_1(\theta,\phi)}^{\psi_2(\theta,\phi)} f(\rho\sin\phi\cos\theta\,,\rho\sin\phi\sin\theta\,,\rho\cos\phi)\rho^2\sin\phi\,d\rho d\theta d\phi$$

$$= \iiint_S f\,\rho^2\sin\phi\,d\rho d\theta d\phi$$

EX 3

Find $\iiint_S f(x,y,z)dV$ for $f(x,y,z) = x^2 + y^2$ on $S = \{(x,y,z) \mid x\text{^}2 + y\text{^}2 + z\text{^}2 \leq 1\}$

EX 4

Find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 16$, outside the cone, $z = \sqrt{x^2 + y^2}$, and above the xy-plane.