Math 2210 #23
Triple Integrals

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$A = \int_a^b f(x)dx$ Measures 2-D space (signed area) under a curve above the x-axis.

$V = \iint_S f(x,y)dA$ Measures 3-D space (signed volume) under a surface above the xy-plane.

We predict that $\iiint f(x,y,z)dV$ measures 4-D space (signed) under a "hyper" sufface "above" the xyz-"hyper-plane".

$$\iiint_S f(x,y,z)dV = \int_{a_1}^{a_2} \int_{\varphi_1(x)}^{\varphi_2(x)} \int_{\psi_1(x,y)}^{\psi_2(x,y)} f(x,y,z)dzdydx$$

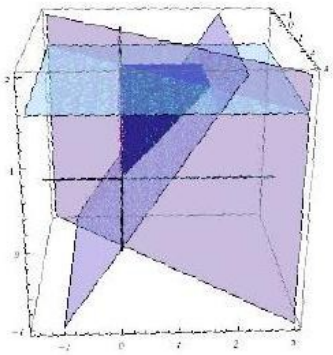
Note: We can't draw anything in 4-D, but we can draw the region S in 3-D (domain space is now 3-D).

EX 1

Write an iterated integral for $\iiint_S (y + z + 1)dV$ where $S = \{(x, y, z) \mid x \in [0,1], y \in [2,5], z \in [1,4]\}$.

EX 2

Evaluate $\int_0^{\pi/2} \int_0^z \int_0^y \sin(x + y + z)dxdydz$.

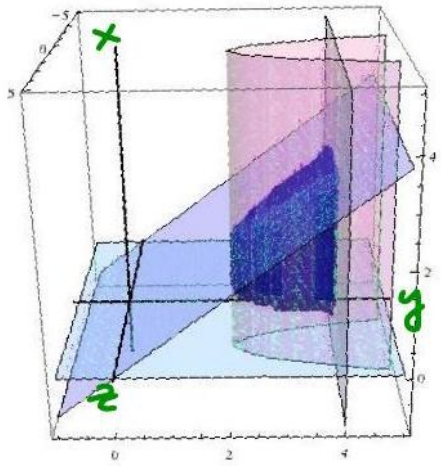


EX 3

Write an iterated integral for $\iiint_S f(x,y,z)dV$
where S is the region in the first octant bounded by the surface $z = 9 - x^2 - y^2$ and the coordinate planes.

EX 4

Find the volume of the solid in the first octant bounded by the hyperbolic cylinder $y^2 - 64z^2 = 4$ and the plane $y = x$ and $y = 4$.



EX 5

Find the volume of the tetrahedron with vertices at (0,0,0), (0,0,3), (0,4,0), and (2,0,0).