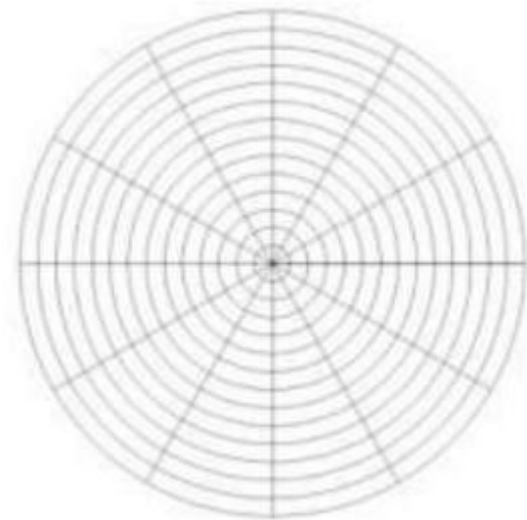


Math 2210 #21

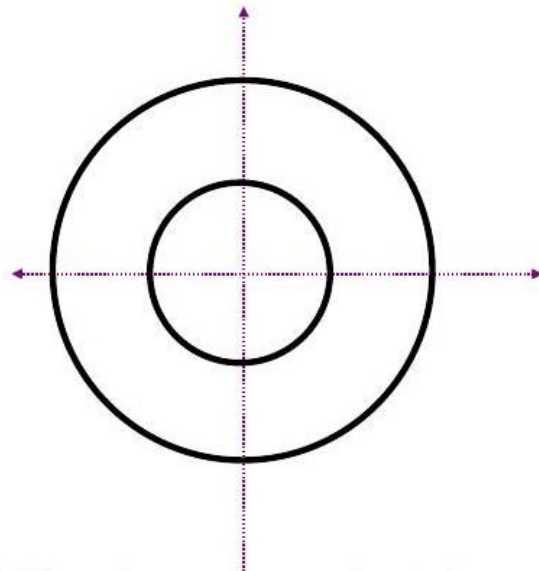
Double Integrals in Polar Coordinates

Double Integrals in Polar Coordinates

Rather than finding the volume over a rectangle (for Cartesian Coordinates), we will use a "polar rectangle" for polar coordinates.



$$A_{\text{sector}} = \pi r^2 \frac{\Delta\theta}{2\pi} = \frac{1}{2} \Delta\theta r^2$$



Area of a polar rectangle

Why do we want to integrate polar coordinates?

EX 1

Find the area of the given region S by calculating

$$A = \iint_S dA = \iint_S r dr d\theta$$

1a)

S is the smaller region bounded by $\theta = \pi/6$ and $r = 4\sin \theta$.

EX 1

(cont'd) Find the area of the given region S by calculating

$$\iint_S r dr d\theta$$

1b)

S is the region outside the circle $r = 2$ and inside the lemniscate $r^2 = 9\cos(2\theta)$.

EX 2

Evaluate using polar coordinates.

2a)

$\iint_S y dA$ where S is the first quadrant polar rectangle inside $x^2 + y^2 = 4$ and outside $x^2 + y^2 = 1$.

2b)

$$\iint_S (x^2 + y^2) dA$$

EX 2

(cont'd) Evaluate using polar coordinates.

2c)

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \sin(x^2 + y^2) dx dy$$