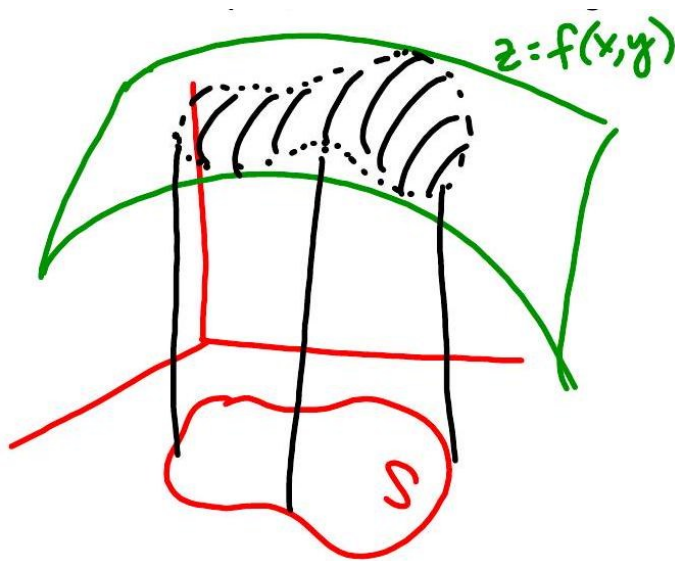


Math 2210 #20

Double Integrals Over Non-rectangular Regions

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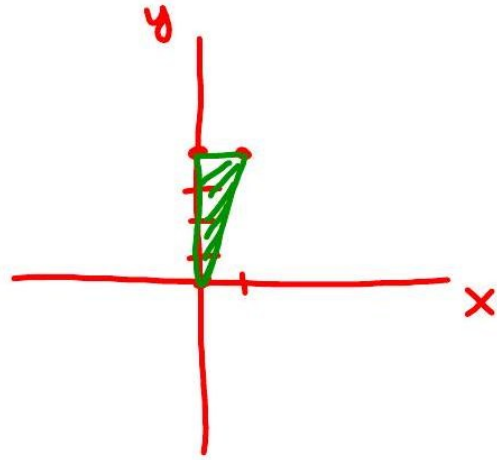
What if the region we're integrating over is not a rectangle, but a simple, closed curve region instead?



$$\begin{aligned} V &= \iint_S f(x, y) dA \\ &= \int_a^b \int_{\phi_1(x)}^{\phi_2(x)} f(x, y) dy dx \\ &= \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) dx dy \end{aligned}$$

EX 1

Find $\iint_S 5dA$ where S is the triangle with vertices at $(0,0)$, $(0,4)$, and $(1,4)$.

**EX 2**

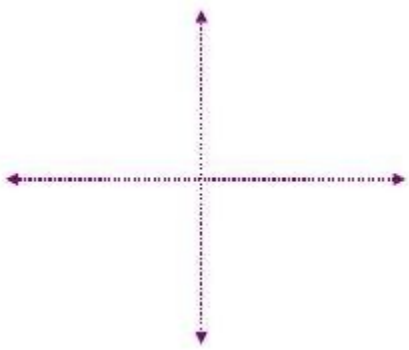
Evaluate $\iint_S x dA$ where S is the region between $y = x$ and $y = x^2$ in the first octant.

EX 3

Write these integrals as iterated integrals with the order of integration switched.

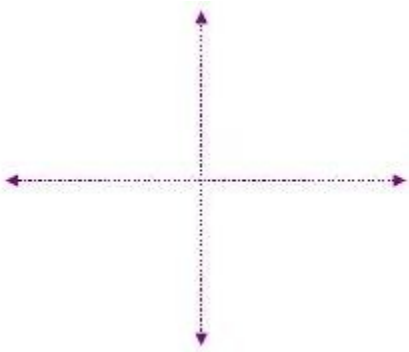
3a)

$$\int_0^2 \int_{y^2}^{2y} f(x, y) dx dy$$



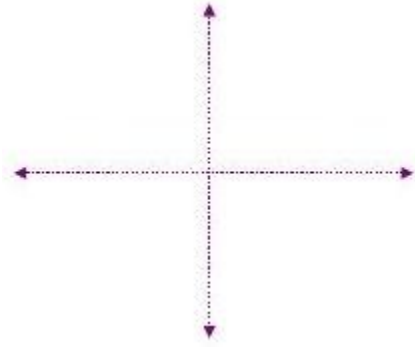
3b)

$$\int_{1/2}^1 \int_{x^3}^x f(x, y) dy dx$$



3c)

$$\int_0^1 \int_{-y}^y f(x,y) dx dy$$



EX 4

Evaluate

4a)

$$\int_1^5 \int_0^x \frac{3}{x^2 + y^2} dy dx$$

4b)

$$\int_{\pi/6}^{\pi/2} \int_0^{\sin \theta} 6r \cos \theta dr d\theta$$

EX 5

Find the volume of the solid bounded by the parabolic cylinder $x^2 = 4y$ and the planes $z = 0$ and $5y + 9z - 45 = 0$.