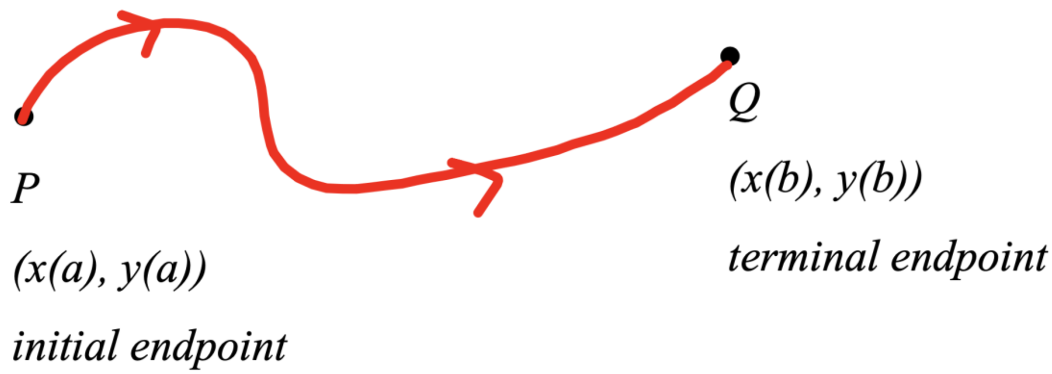
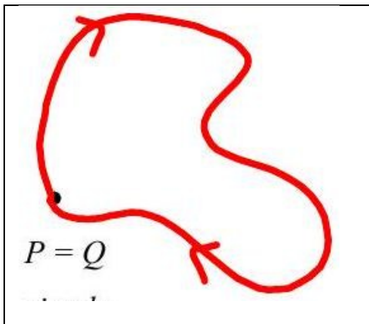
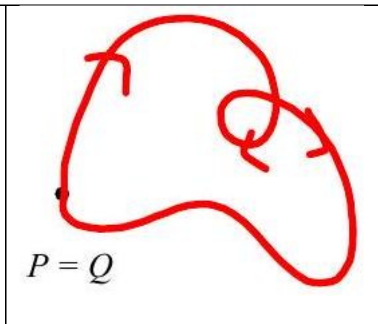
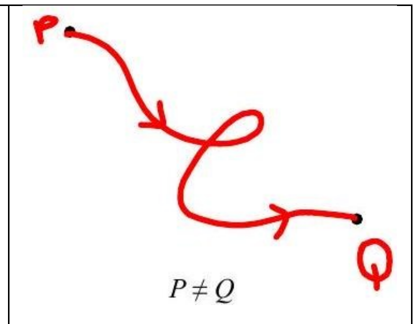
Math 2210 #1
Parametric Representations of Plane Curves

A plane curve is a 2 -dimensional curve given by

$$x = f(t)\ y = g(t)\ t \in I$$

where f and g are continuous functions on the interval I, $[a, b]$. t is the parameter. $t \in [a, b]$



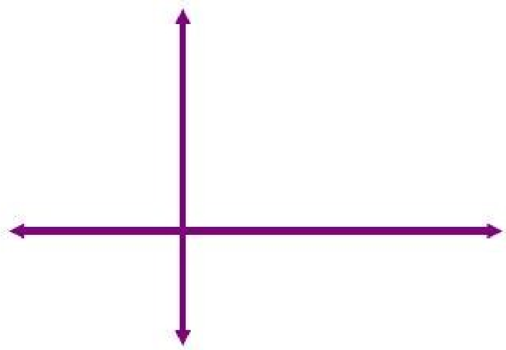
		
simple, closed curve	not simple, closed curve	not simple, not closed curve

It can be hard to recognize the shape of a curve when given parametically. Sometimes it is possible to eliminate the parameter.

EX 1

Eliminate the parameter and sketch this curve.

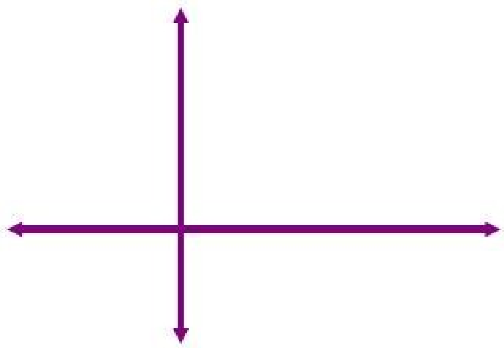
$$x = t - 3 \qquad y = \sqrt{t} \qquad 0 \le t \le 8$$



EX 2

Eliminate the parameter t, graph the curve and tell if it is simple and closed.

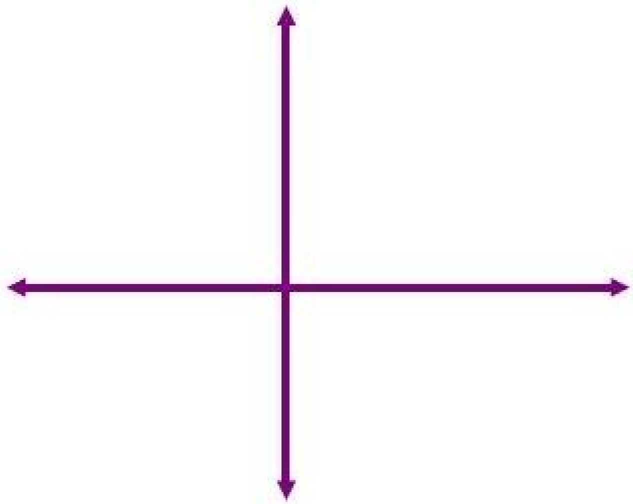
$$x = \sqrt{t-3} \qquad y = \sqrt{4-t} \qquad 3 \le t \le 4$$



EX 3

Eliminate the parameter θ, graph the curve and tell if it is simple and closed.

$$x = \sin\,\theta \qquad y = 2\cos^2(2\theta) \qquad \theta \in \mathbb{R}$$



Theorem A

Let f and g be continuously differentiable with $f'(t) \neq 0$ on $t \in (\alpha, \beta)$.
Then the parametric equations $x = f(t)$ and $y = g(t)$ define y as a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} \quad \text{where } y' = \frac{dy}{dx}$$

EX 4

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (without eliminating the parameter.)

4a)

$x = \sqrt{3}\theta^2 \quad y = -\sqrt{3}\theta^3 \quad \theta \neq 0$

4b)

$$x = \frac{2}{1+t^2} \qquad y = \frac{2}{t(1+t^2)} \qquad t \neq 0$$

Length of a curve: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

EX 5

Find the length of the curve given by

$$x = \sin\ \theta - \theta\cos\ \theta\ \theta \in [\pi/4, \pi/2]$$
$$y = \cos\ \theta + \theta\sin\ \theta$$