Math 2210 #16
Tangent Planes

Tangent Planes

We already dealt with tangent planes to surfaces of form $z = f(x, y)$.
Now, we will find tangent planes to surfaces of form $F(x, y, z) = k$, i.e. a surface represented by any equation in three variables.

Definition

Let $F(x, y, z) = k$ be a surface, F, differentiable at $P(x_0, y_0, z_0)$ with $\nabla F(x_0, y_0, z_0) \neq \vec{0}$.
Then the plane through P and perpendicular to $\nabla F(x_0, y_0, z_0)$ is called the tangent plane to the surface at P.

Theorem

For surface $F(x, y, z) = k$, the equation of the tangent plane at (x_0, y_0, z_0) is

$$\nabla F(x, y, z) \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$\Leftrightarrow$$

$$\boldsymbol{F_x}(x_0, y_0, z_0)(\boldsymbol{x} - \boldsymbol{x}_0) + \boldsymbol{F_y}(x_0, y_0, z_0)(y - y_0) + \boldsymbol{F_z}(\boldsymbol{x}_0, y_0, z_0)(z - z_0) = \mathbf{0}.$$

EX 1

Find the equation of the tangent plane to $8x^2 + y^2 + 8z^2 = 16$ at $\left(1,2,\frac{\sqrt{2}}{2}\right)$.

EX 2

Find parametric equations of the line that is tangent to the curve of intersection of these surfaces at the point (1,2,2).

$$f(x, y, z) = 9x^2 + 4y^2 + 4z^2 - 41 = 0$$
$$g(x, y, z) = 2x^2 - y^2 + 3z^2 - 10 = 0$$

Definition

Let $z = f(x, y), f$ is differentiable function, dx and dy (differentials) are variables. dz (also called total differential of f) is

$$dz = df(x, y) = f_x(x, y)dx + f_y(x, y)dy = \nabla f \cdot \langle dx, dy \rangle$$

EX 3

Use differentials to approximate the change in z as (x, y) moves from P to Q. Also find Δz.

$z = x^2 - 5xy + y$ $P(2,3)$ $Q(2.03,2.98)$

EX 4

Use differentials to find the approximate amount of copper in the four sides and bottom of a rectangular copper tank that is 6 feet long, 4 feet wide and 3 feet deep inside, if the sheet-copper is $\frac{1}{4}$ inch thick.

EX 5

A piece of cable (cylindrical) that measures 2 meters long with a radius of 2 centimeters is thought to have measurement error as large as 5 millimeters for each of the height and radius measurements. Estimate the error in the volume measurement.