Math 2210 #13
Differentiability/Gradient

Differentiability

For a function of one variable, the derivative gives us the slope of the tangent line, and a function of one variable is differentiable if the derivative exists. For a function of two variables, the function is differentiable at a point if it has a tangent plane at that point. But existence of the first partial derivatives is not quite enough, unlike the one-variable case.

Theorem

If $f(x, y)$ has continuous partial derivatives $f_x(x, y)$ and $f_y(x, y)$ on a disk D whose interior contains (a, b), then $f(x, y)$ is differentiable at (a, b).

Theorem

If f is differentiable at (a, b), then f is continuous at (a, b).

Gradient of f

$$\nabla f(p) = \nabla f(a, b) = \langle f_x(a, b), f_y(a, b) \rangle = f_x(a, b)\hat{\imath} + f_y(a, b)\hat{\jmath}$$

for a function, $z = f(x, y)$.
(Note: This gradient lives in 2-D space, but it is the gradient of a function whose graph is 3-D.)

Properties of Gradient Operator

p is the input point (a, b).

$$\nabla[f(p) + g(p)] = \nabla f(p) + \nabla g(p)$$
$$\nabla[\alpha f(p)] = \alpha \nabla f(p), \alpha \in \Re$$
$$\nabla[f(p)g(p)] = f(p)\nabla g(p) + \nabla f(p)g(p)$$

EX 1

Find the gradient of f.

1a)

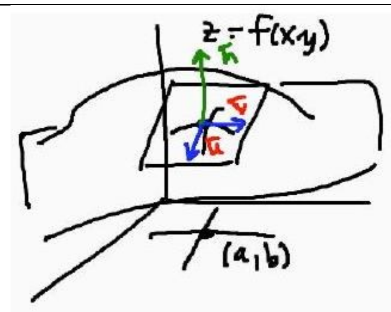
$f(x, y) = x^3 y - y^3$

1b)

$f(x, y) = \sin^3 (x^2 y)$

1c)

$f(x, y, z) = xz\ln (x + y + z)$

Curves in 2-D	Surfaces in 3-D
Remember the equation of the tangent line to a 2-D curve.	

EX 2

For $f(x, y) = x^3y + 3xy^2$, find the equation of the tangent plane at $(a, b) = (2, -2)$.

EX 3

Find the equation of the tangent "hyperplane" to $f(x, y, z)$ at the point (a, b, c).

$$f(x, y, z) = xyz + x^2 \ (a, b, c) = (2,0,-3)$$

EX 4

Find all domain points (x, y) at which the tangent plane to the graph of $z = x^3$ is horizontal.