

Math 2210 #12

Limits and Continuity

Review from Calculus 1

Definition: Continuity at a Point

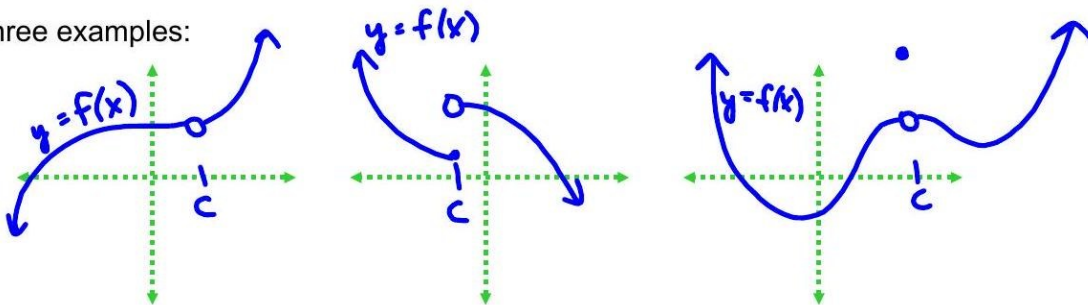
Let f be defined on an open interval containing c . We say that f is continuous at c if

$$\lim_{x \rightarrow c} f(x) = f(c)$$

This indicates three things:

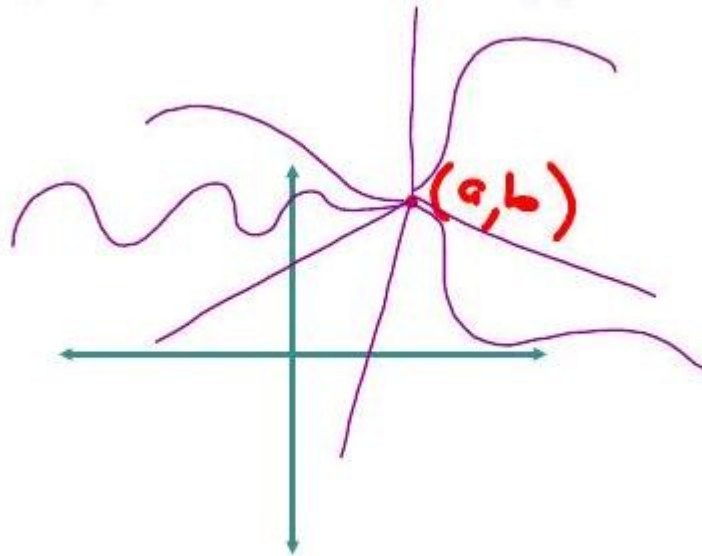
1. The function is defined at $x = c$.
2. The limit exists at $x = c$.
3. The limit at $x = c$ needs to be exactly the value of the function at $x = c$.

Three examples:



Limits and Continuity

Intuitively, $\lim_{(x,y) \rightarrow (a,b)} f(x,y) = L$ means that as the point (x,y) gets very close to (a,b) , then $f(x,y)$ gets very close to L . When we did this for functions of one variable, it could approach from only two sides or directions (left or right). Now we can approach (a,b) from infinitely many directions.



Definition of Limit

$\lim_{(x,y) \rightarrow (a,b)} f(x,y) = L$ means that for all $\varepsilon > 0$ there exists a corresponding $\delta > 0$ such that $|f(x,y) - L| < \varepsilon$ provided that $0 < |(x,y) - (a,b)| < \delta$. We can make $f(x,y)$ as close as we'd like to L by choosing (x,y) sufficiently close to (a,b) .

$$|(x,y) - (a,b)| = \sqrt{(x-a)^2 + (y-b)^2}$$

EX 1

Find $\lim_{(x,y) \rightarrow (-2,1)} (xy^3 - xy + 3y^2)$.

EX 2

Find

$$\lim_{(x,y) \rightarrow (0,0)} \frac{1 + xy}{\cos(xy)}$$

EX 3

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\tan(x^2 + y^2)}{x^2 + y^2}$$

EX 4

Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy+y^3}{x^2+y^2}$ does not exist.

EX 5

Find the limits.

5a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^3}{x^2+y^2}$$

Hint: Use polar coordinates.

5b)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x}{x^2 + y^2}$$

5c)

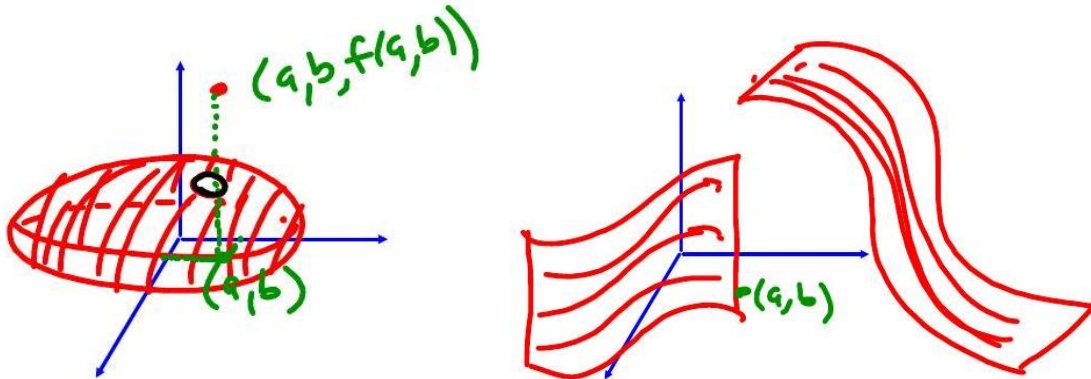
$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$$

Continuity

A function $f(x, y)$ is continuous at (a, b) if $f(a, b) = \lim_{(x, y) \rightarrow (a, b)} f(x, y)$

This indicates three things:

- the function is defined at (a, b) ,
- the limit of f as $(x, y) \rightarrow (a, b)$ exists, and
- the limit of f at (a, b) is exactly the same as $f(a, b)$.



Composition of Functions

If a function, g , of two variables is continuous at (a, b) and a function, f , of one variable is continuous at $g(a, b)$, then

$(f \circ g)(x, y) = f(g(x, y))$ is continuous at (a, b) .

EX 6

Show that $f(x, y) = \sin(x^2 - 4xy)$ is continuous everywhere.

EX 7

Determine where $f(x, y) = \ln(1 - x^2 - y^2)$ is continuous.

EX 8

Is $f(x, y) = \begin{cases} \frac{\sin(xy)}{xy} & \text{if } xy \neq 0 \\ 1 & \text{if } xy = 0 \end{cases}$ continuous everywhere?