Math 2210 #11
Partial Derivatives

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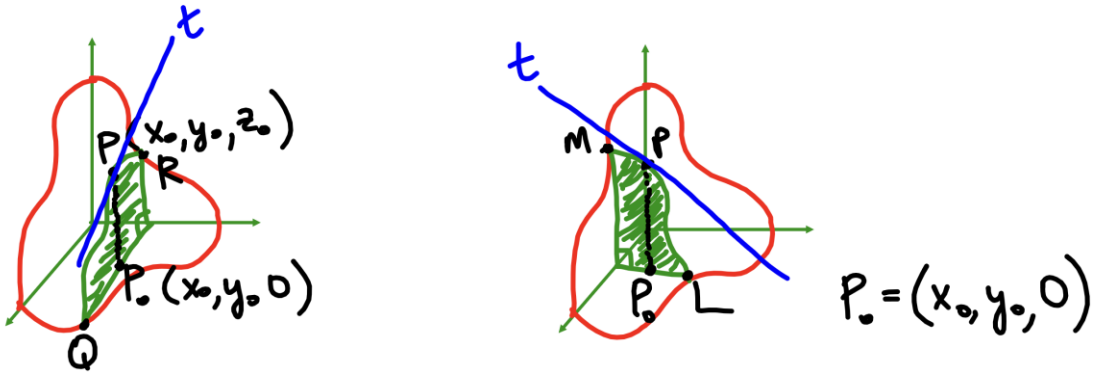

Consider the same surface cut by two different planes.
In **a** it is cut by $y = y_0$,
in **b** it is cut by $x = x_0$.

The curve of intersection in a goes through plane RPQ and in b through plane MPL.

Each of those curves has a tangent line associated with it at point P.

Each tangent line has a steepness associated with it and that should make us think about what?

Since our function is now a function of two variables (rather than one), we can only take the partial derivative with respect to one of the variables.

$$f_x(x_0, y_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$
$$f_y(x_0, y_0) = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

EX 1

Find $f_x(0,3)$ and $f_y(0,3)$ if $f(x, y) = 3x^2y^2 + 4y^3 - 5$.

Notation

If $z = f(x, y)$, then

$$f_x(x, y) = \frac{\partial z}{\partial x} = \frac{\partial f(x, y)}{\partial x} \quad partial\ derivative\ of\ f\ with\ respect\ to\ x$$
$$f_y(x, y) = \frac{\partial z}{\partial y} = \frac{\partial f(x, y)}{\partial y} \quad partial\ derivative\ of\ f\ with\ respect\ to\ y$$

EX 2

$$If\ z = x^2y + cos\ (xy) - 2,\ find\ \frac{\partial z}{\partial x}\ and\ \frac{\partial z}{\partial y}$$

EX 3

Find the 'slope' of the tangent line to the curve of intersection of this surface

$3z = \sqrt{36 - 9x^2 - 4y^2}$
and the plane $x = 1$
at the point $(1, -2, \sqrt{11}/3)$.

The 'slope' here refers to the change in z over the change in y.

EX 4

The temperature in degrees celsius on a metal plate in the xy-plane is given by $T(x, y) = 4 + 2x^2 + y^3$. What is the rate of change of temperature with respect to distance (in feet) if we start moving from (3,2) in the direction of the y -axis?

Higher Order Partial Derivatives

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} \qquad f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2}$$

$$f_{xy} = (f_x)_y = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} \quad f_{yx} = (f_y)_x = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y}$$

EX 5

Find all four second partial derivatives for $f(x, y) = (x^3 + y^2)^5$.

EX 6

Find all four second partial derivatives for $f(x, y) = \tan^{-1}(xy)$.

EX 7

For $f(x,y,z) = xy^2 - \frac{2x}{yz} + 3z^3x$, find f_x, f_y, f_z, f_{xz} and f_{yy}.