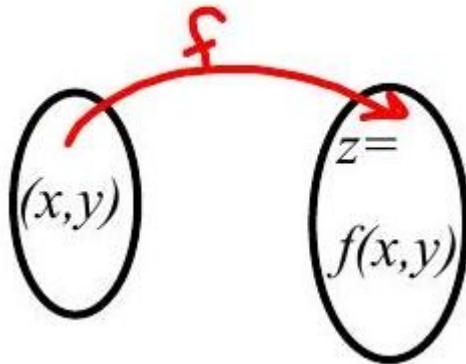


Math 2210 #10

Functions of Two or More Variables

A real-valued function of 2 variables takes two real input values and returns one real output value.

e.g. $f(x, y) = x^2 + 3y^2$ or $g(x, y) = \sqrt{xy} + 2x^3$.



independent variables \Rightarrow

dependent variable \Rightarrow

domain \Rightarrow

range \Rightarrow

EX 1

$f(x, y) = \frac{y}{x} + xy$, find

1a)

$f(1, 2)$

1b)

$f(a, a)$

1c)

$f\left(\frac{1}{x}, x^2\right)$

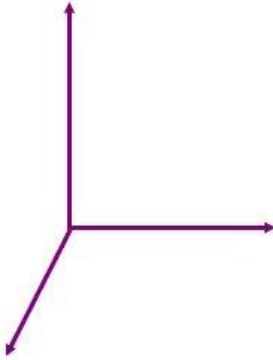
1d)

What is the domain of f ?

The graph of a function of 2 variables is a 3D surface (usually). Since it is a function, then to each output, z , there can only be one (x, y) from the domain. Graphically, this means that each line perpendicular to the xy -plane intersects the surface in at most one point.

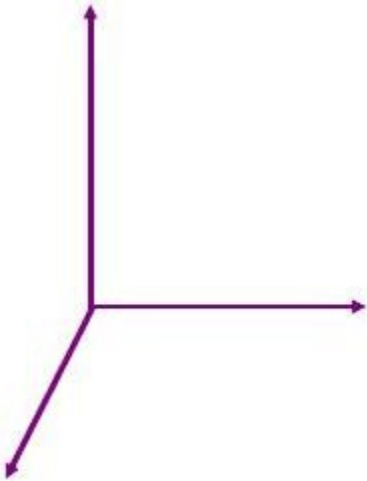
EX 2

Sketch the graph of $f(x, y) = 6 - x^2$.



EX 3

Sketch the graph of $f(x, y) = \sqrt{16 - 4x^2 - y^2}$.

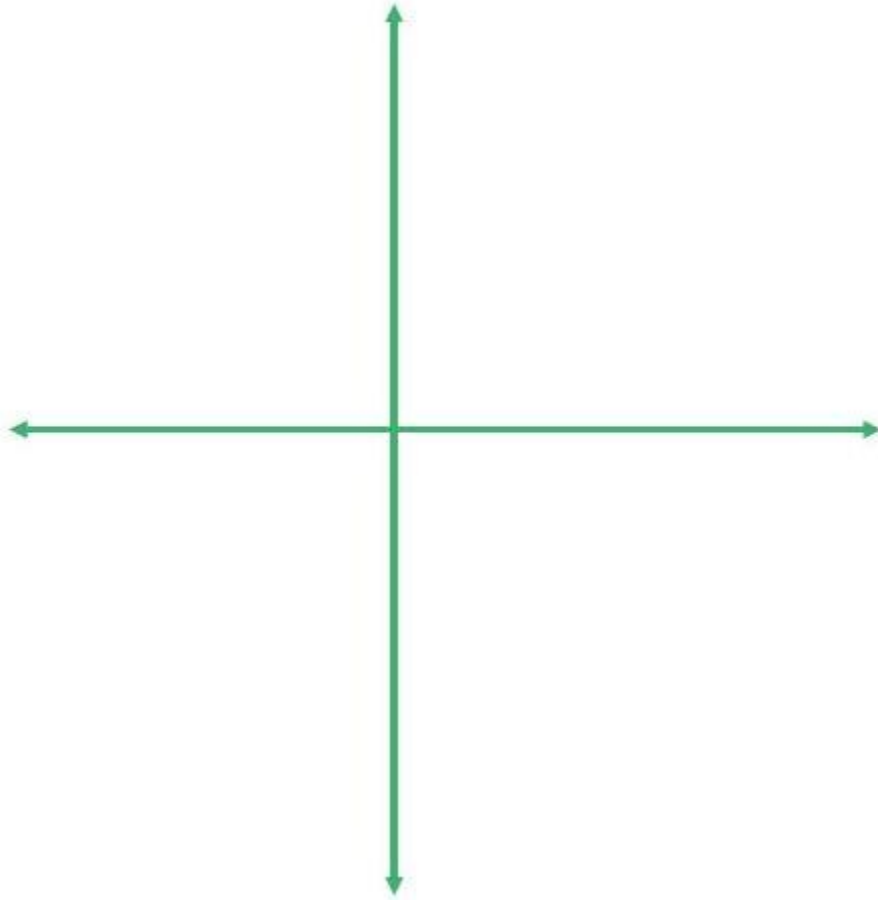


Level Curves \Rightarrow Projection of intersecting curves (with surface and planes $z = c$, c is real) onto the xy -plane.

Contour Map \Rightarrow a collection of level curves.

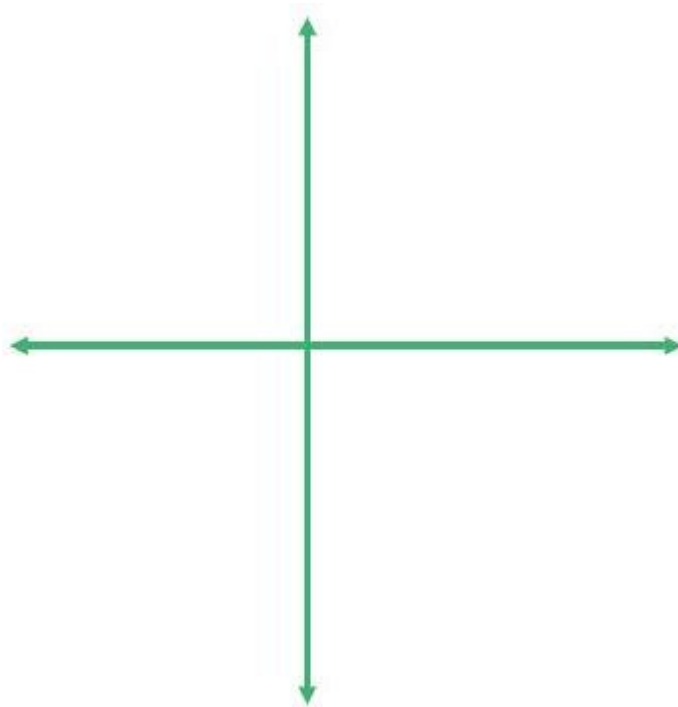
EX 4

Sketch level curves at $z = -1, 0, 1, 4, 9$ for $z = 1/4x^2 + y^2$.



EX 5

Sketch level curves at $z = -4, -1, 0, 1, 4$ for $z = y^2 - x^2$.

**EX 6**

Find the domain for $f(x, y, z) = \sqrt{x^2 + y^2 - z^2 - 9}$.