Math 1220 #2

Inverse Functions

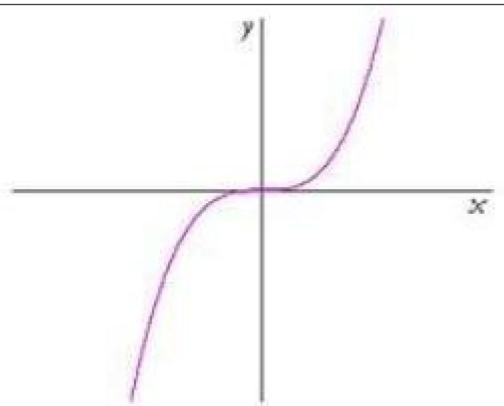
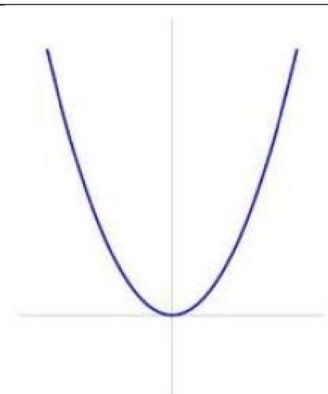
Inverse Functions

If $f(x)$ and $f^{-1}(x)$ are inverse functions:

- $f(x)$ must be one-to-one,
  I.e. The inverse exists when we can get back to an x given a y.
  The horizontal line test may be used.
- If (a,b) is on $f(x)$, then (b,a) is on $f^{-1}(x)$.
- $f(f^{-1}(x)) = f^{-1}(f(x)) = x$
- The domain of $f(x)$ becomes the range of $f^{-1}(x)$
- The range of $f(x)$ becomes the domain of $f^{-1}(x)$

Note: $f^{-1}(x)$ is not the reciprocal, $\frac{1}{f^{-1}(x)}$

Let's look at two functions:

$f(x) = x^3$	$f(x) = x^2$
	

If we don't have a graph, how can we algebraically test if a function has an inverse?

Theorem A

If f is strictly monotonic on its domain, then f has an inverse.

EX 1

Show that this function has an inverse, but do not find it.

$$f(x) = 3x^7 + 4x^3 + x - 3$$

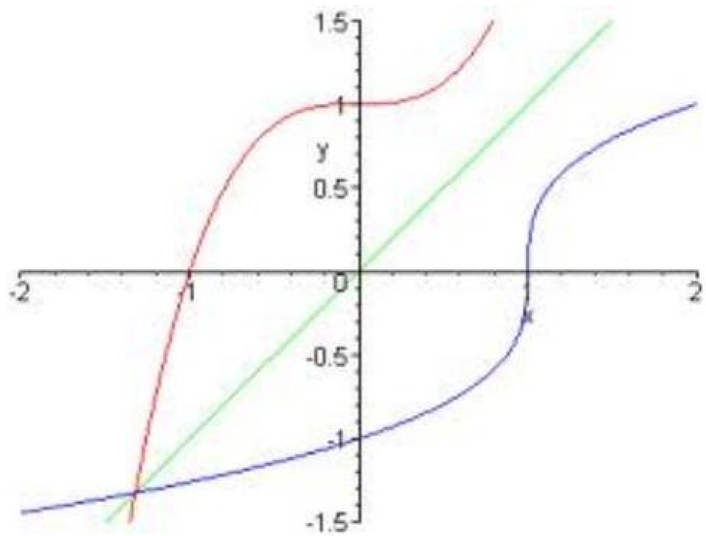
EX 2

Explore whether this function has an inverse. If not, can we restrict the domain so it does? If so, find $f^{-1}(x)$.
$f(x) = x^2 - 4$

EX 3

Find $f^{-1}(x)$ for this function and check your work.

$$y = \frac{2x - 1}{3 + 5x}$$



The graph of $f^{-1}(x)$ is $f(x)$ reflected across the line $y = x$.

Notice the slope at (d, c) and the slope at (c, d).

$$(f^{-1})'(d) = \frac{1}{f'(c)}$$

Theorem B: Inverse Function Theorem

If f is differentiable, strictly monotonic on an interval and
$f'(x) \neq 0$ at some x on the interval,
then $f^{-1}(x)$ is differentiable at a corresponding point
in the range of f and $(f^{-1})'(y) = \frac{1}{f'(x)}$

$$\frac{dx}{dy} = \frac{1}{dy/dx}$$

EX 4

Find $(f^{-1})'(2)$ using theorem B. $f(x) = x^5 + 5x - 4$