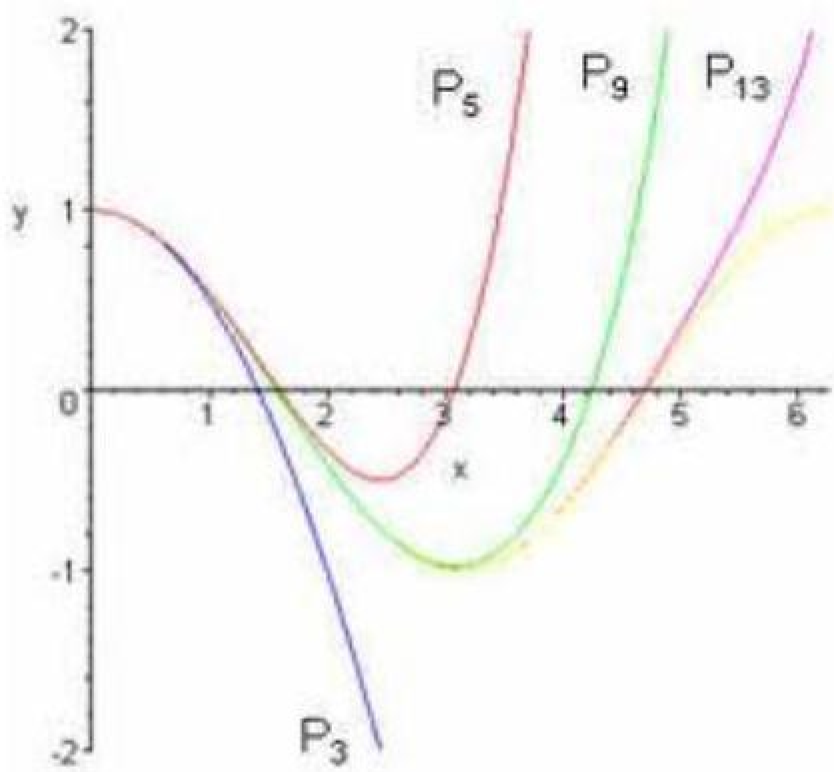
Math 1220 #26
The Taylor Approximation to a Function

Taylor Approximations to a Function

Many math problems that occur in applications cannot be solved exactly, like $\int_0^b \sin(x^2)\,dx$. We need to approximate them.

Taylor Polynomial of order n (based at a)

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$



EX 1

For $f(x) = e^{-3x}$, find the Maclaurin polynomial of order 4 and approximate $f(0.12)$.

Lagrange Error for Taylor Polynomials

We know $f(x) = P_n(x) + R_n(x)$.

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

$c \in (a, x)$ if $a < x$ or $c \in (x, a)$ if $x < a$

EX 2

Find the error in estimating $f(0.12)$ in the last example, $f(x) = e^{-3x}$.

EX 3

Find a good bound for the maximum value of $\left|\frac{4c}{c+4}\right|$ given $c \in [0,1]$.

EX 4

Find a good bound for the maximum value of $\left|\frac{c^2-c}{cos\ c}\right|$ given $c \in [0, \pi/4]$.

EX 5

Find n such that the Maclaurin polynomial for $f(x) = e^x$ has $f(1)$ approximated to five decimal places, i.e. $|R_n(1)| \leq 0.000005$.