Math 1220 #23
Power Series

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Consider a series of functions instead of constants.

Power Series in x

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots \quad a_0 = a_0 x^0$$

EX 1

When does this power series converge, i.e., for what x-values?

$$\sum_{n=0}^{\infty} a x^n \begin{Bmatrix} a \in \Re \\ a \neq 0 \end{Bmatrix}$$

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Theorem

The convergence set for a power series $\sum_{n=0}^{\infty} a_n x^n$ is always an interval of one of these three types.

1. The single point at $x = 0$.
2. An interval, $(-R, R), [-R, R], [-R, R)$, or $(-R, R]$
3. $(-\infty, \infty)$

The radius of convergence is $0, R$, or ∞, respectively.

EX 2

Find the convergence set for $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \cdots$.

EX 3

Find the convergence set for $1 + x + \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} + \frac{x^4}{\sqrt{4}} + \cdots$.

Power Series in $(x - c)$

$$\sum_{n=0}^{\infty} a_n(x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \cdots$$

Convergence set:

1. The single point at $x = c$
2. An interval, $(\ c - R, c + R\ )$ (may include endpoints)
3. $(-\infty, \infty)$

EX 4

Find the convergence set for $\frac{x-3}{2} + \frac{(x-3)^2}{2^2} + \frac{(x-3)^3}{2^3} + \cdots$.