

Math 1220 #22

Alternating Series, Absolute Convergence and Conditional Convergence

In an Alternating Series, every other term has the opposite sign.

$$a_1 - a_2 + a_3 - a_4 + a_5 - \dots$$

AST (Alternating Series Test)

Let $a_1 - a_2 + a_3 - a_4 + \dots$ be an alternating series such that $a_n > a_{n+1} > 0$, then the series converges.

The error made by estimating the sum, S_n is less than or equal to a_{n+1} , i.e.,
 $E = |S - S_n| \leq a_{n+1}$.

EX 1

Does an Alternating Harmonic Series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

EX 2

Does this series diverge or converge? What is the error estimate made when approximating the sum using S_6 ?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{n^2 + 1}$$

Absolute Convergence

If $\sum |u_n|$ converges, then $\sum u_n$ converges.

EX 3

Does $2 + \frac{2}{2^3} + \frac{2}{3^3} - \frac{2}{4^3} + \frac{2}{5^3} + \frac{2}{6^3} + \frac{2}{7^3} - \frac{2}{8^3} + \dots$ converge or diverge?

Absolute Ratio Test

Let $\sum a_n$ be a series of nonzero terms and suppose $\lim_{n \rightarrow \infty} \frac{|u_{n+1}|}{|u_n|} = \rho$.

- i) if $\rho < 1$, the series converges absolutely.
- ii) if $\rho > 1$, the series diverges.

iii) if $\rho = l$, then the test is inconclusive.

EX 4

Show $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{e^n}$ converges absolutely.

Conditional Convergence

$\sum u_n$ is conditionally convergent if $\sum u_n$ converges but $\sum |u_n|$ does not.

EX 5

Classify as absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1} + \sqrt{n}}$$

Rearrangement Theorem

The terms of an absolutely convergent series can be rearranged without affecting either the convergence or the sum of the series.