Math 1220 #17
Improper Integrals: Infinite Integrands

Look at $\int_{-1}^{2} \frac{1}{x^4} dx$. Can we just do the integral?

Definition

Let f(x) be continuous on [a, b] and

$$\lim_{x \to b^-} |f(x)| = \infty \Rightarrow \int_a^b f(x)dx = \lim_{t \to b^-} \int_a^t f(x)dx$$

if the limit exists and is finite, otherwise it diverges.

EX 1

$$\int_1^3 \frac{dx}{(x-1)^{4/3}}$$

EX 2

$$\int_0^9 \frac{dx}{\sqrt{9-x}}$$

EX 3

$$\int_0^1 \frac{1}{x^p} dx, p \geq 1$$

Definition

If f is continuous on $[a, b]$ except at $x = c$ where $a < b < c$
and $\lim_{x \to c} |f(x)| = \infty$
then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
if both integrals converge. Otherwise it diverges.

EX 4

$$\int_{-5}^{0} \frac{1}{(x+3)^2} dx$$

EX 5

$$\int_{-3}^{1} \frac{5}{(x+2)^{3/5}} dx$$