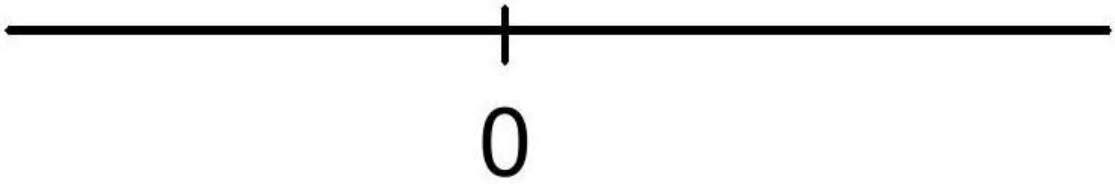
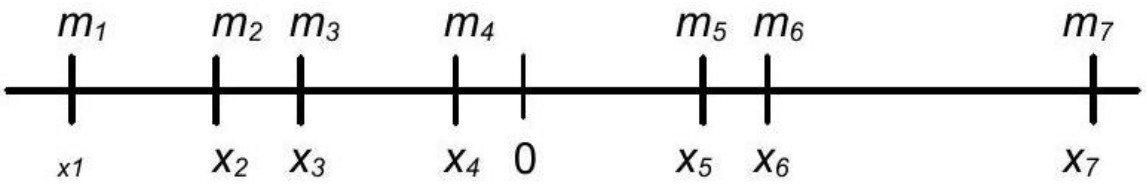
Math 1210 #33
Moments, Center of Mass



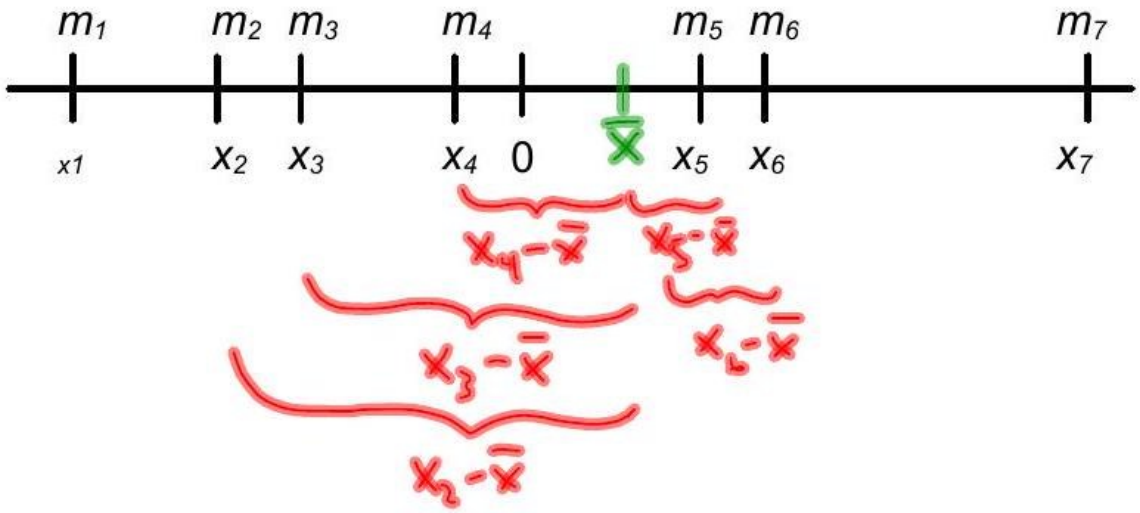
The moment of a particle with respect to a point is the product of mass (m) of the particle with its directed distance (x) from a point. This measures the tendency to produce a rotation about that point.

$$\text{Total moment (M) for a bunch of masses } = \sum_{i=1}^{n} x_i m_i$$

x_i = distance from 0 to particle i

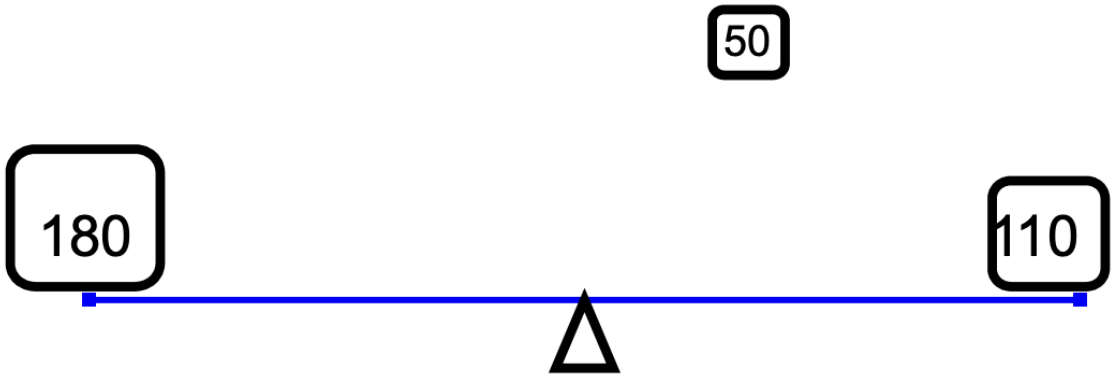


Where does the fulcrum need to be placed to balance? Let's call it $\bar{x}$.



EX 1

John and Mary, weighing 180 lbs and 110 lbs respectively, sit at opposite ends of a 12ft teeter-totter with the fulcrum in the middle. Where should their 50 lb son sit in order for the board to balance?



For a continuous mass distribution along the line (like on a wire):

$$\bar{x} = \frac{M}{m} = \frac{\int_a^b x\delta(x)dx}{\int_a^b \delta(x)dx}$$

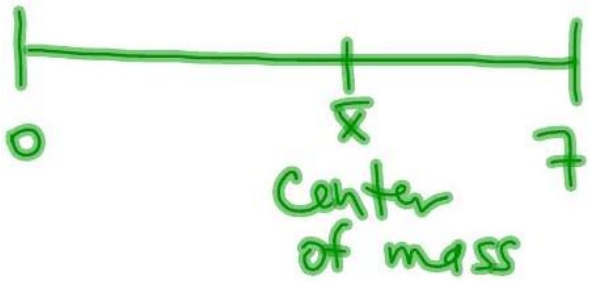
($\delta(x)$ is density function)

Since total mass = $\int_a^b \delta(x)dx$

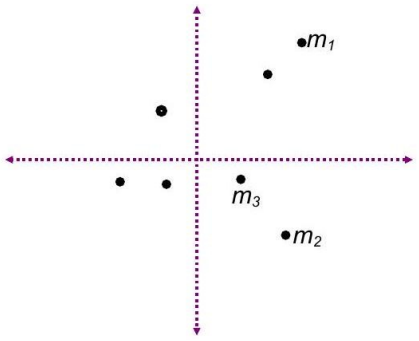
And moment is $\int_a^b x\delta(x)\,dx$

EX 2

A straight wire 7 units long has density $\delta(x) = 1 + x^3$ at a point x units from one end. Find the distance from this end to the center of mass.



Consider a discrete set of 2-d masses.
How do we find the center of mass (the geometric center) $(\bar{x}, \bar{y})$?



$$\bar{x} = \frac{M_y}{m}, \quad \bar{y} = \frac{M_x}{m}$$

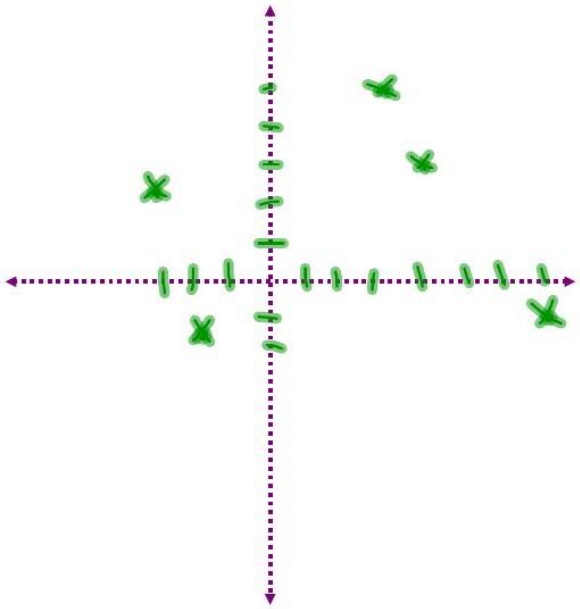
where $M_y = \sum_{i=1}^{n} x_i m_i$

$M_x = \sum_{i=1}^{n} y_i m_i$

total mass $m = \sum_{i=1}^{n} m_i$

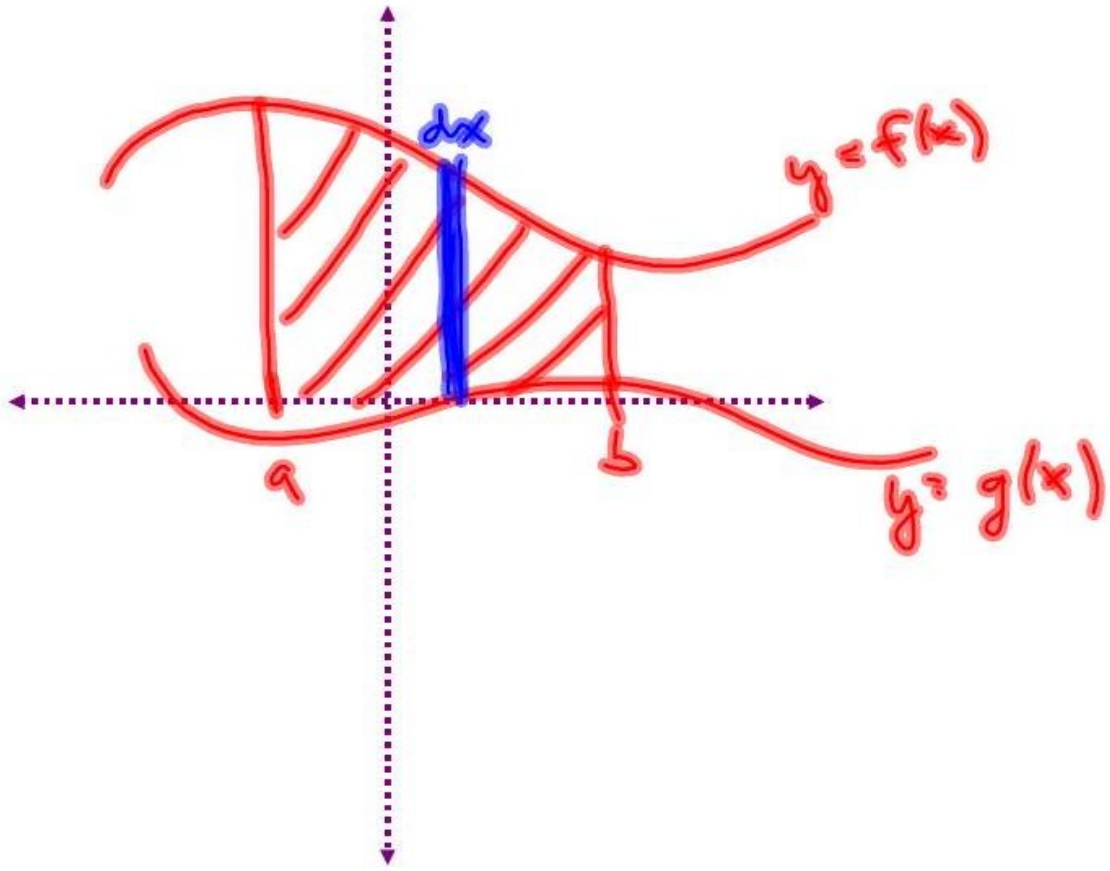
EX 3

The masses and coordinates of a system of particles are given by the following:
$5, (-3,2); 6, (-2,-2); 2, (3,5); 7, (4,3); 1, (7,-1)$. Find the moments of this system with respect to the coordinate axes and find the center of mass.



Now, consider a continuous region 2-d region (a lamina) that has constant (homogeneous) density everywhere. How do we find the center of mass $(\bar{x}, \bar{y})$?

It's still true $\bar{x} = \frac{m_x}{m}, \bar{y} = \frac{m_y}{m}$



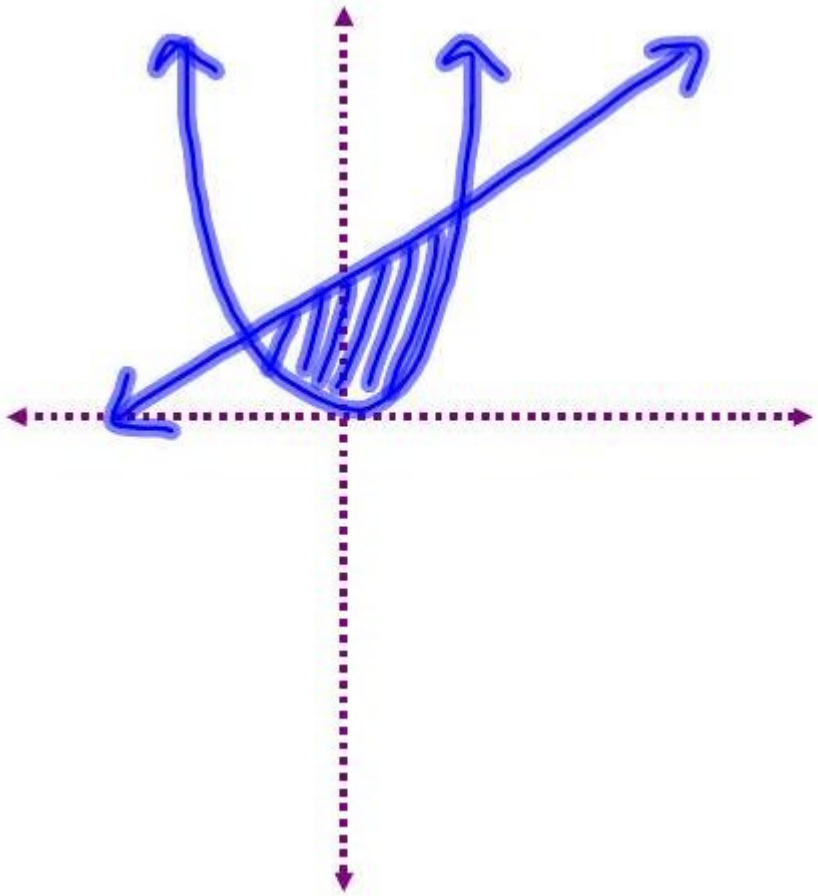
Total mass

$$m = \int_a^b \delta\big(f(x) - g(x)\big)dx$$

(density - area)
δ = density (per area unit)

EX 4

Find the centroid of the region bounded by $y = x^2$ and $y = x + 2$.



Two Children of Equal Mass

One Highschooler and One Elementary Schooler

One Baby and One Cow

One Planet and One Star

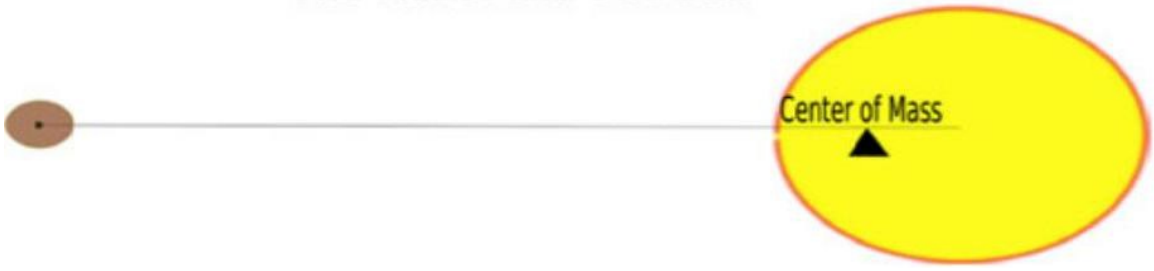


Photo source: Laboratory for Atmospheric and Space Physics, University of Colorado at Boulder