

Math 1210 #31

Length of a Curve and Surface Area

Length of a Plane Curve

A plane curve is a curve that lies in a two-dimensional plane. We can define a plane curve using parametric equations. This means we define both x and y as functions of a parameter.

Parametric equations

Definition

A plane curve is smooth if it is given by a pair of parametric equations $x = f(t)$, and $y = g(t)$, t is on the interval $[a, b]$ where f' and g' exist and are continuous on $[a, b]$ and $f'(t)$ and $g'(t)$ are not simultaneously zero on (a, b) .

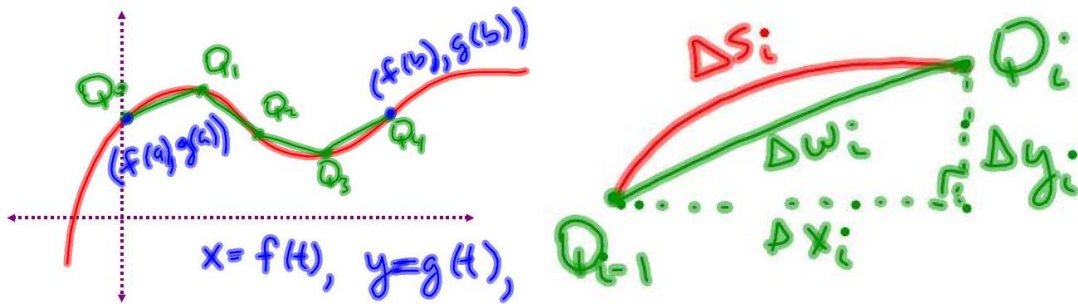
EX 1

Sketch the graph of the curve given by these parametric equations.

$$x = 3t^2 + 2 \quad y = 2t^2 - 1 \quad 1 \leq t \leq 4$$

Arc length

We can approximate the length of a plane curve by adding up lengths of linear segments between points on the curve.



$$L = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2} dt = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

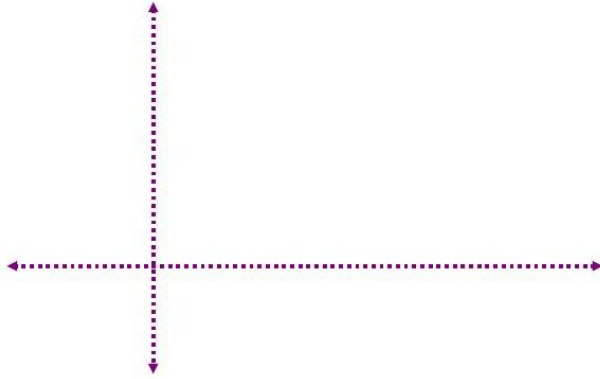
EX 2

Find the circumference of the circle $x^2 + y^2 = r^2$.



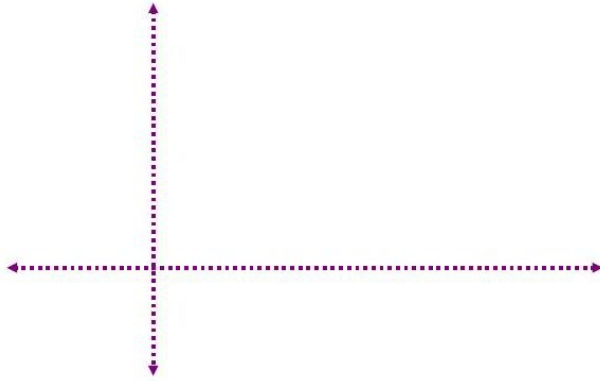
EX 3

Find the length of the line segment on $2y - 2x + 3 = 0$ between $y = 1$ and $y = 3$. Check your answer using the distance formula.



EX 4

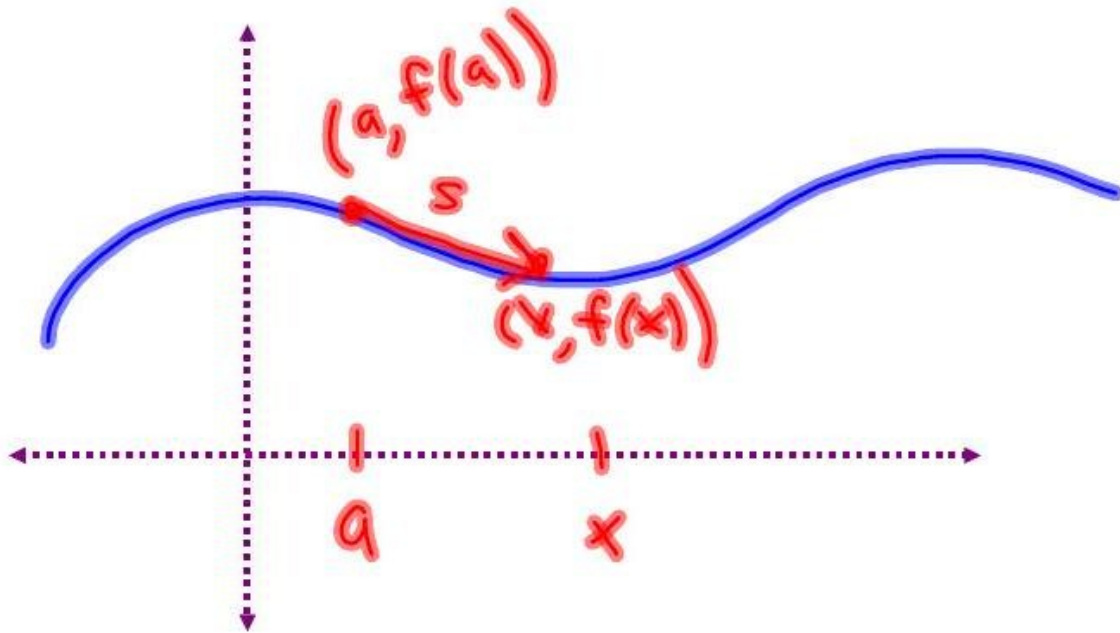
Find the arc length of the curve $f(x) = \sqrt{x}$ from $x = 0$ to $x = 4$.



Surface Area

Differential of Arc Length

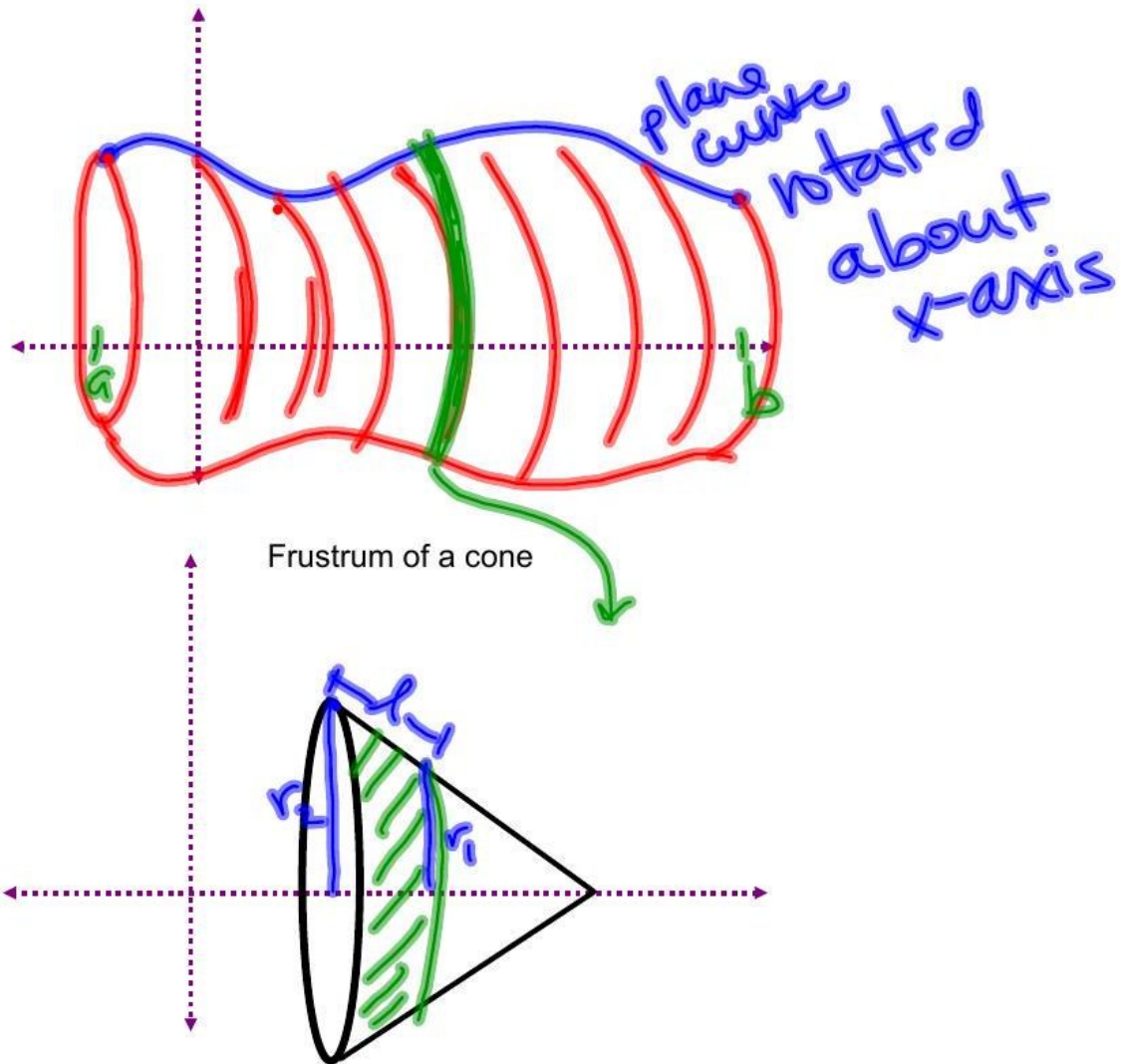
Let $f(x)$ be continuously differentiable on $[a, b]$. Start measuring arc length from $(a, f(a))$ up to $(x, f(x))$, where a is a real number. Then, the arc length is a function of x .
accumulate arc length



$$s(x) = \int_a^x \sqrt{1 + (f'(t))^2} dt$$

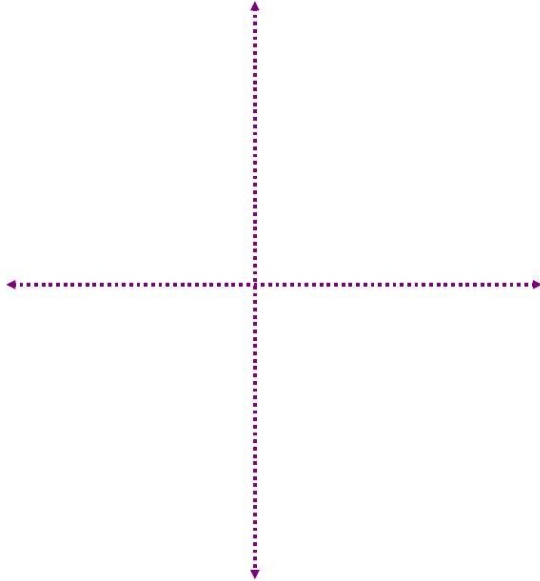
Surface Area of a Surface of Revolution

Rotate a plane curve about an axis to create a hollow three-dimensional solid.
Find the surface area of the solid.



EX 4

Find the area of the surface generated by revolving $y = \sqrt{25 - x^2}$ on the interval $[-2,3]$ about the x -axis.



EX 5

Find the area of the surface generated by revolving $x = 1 - t^2, y = 2t$, on the t -interval $[0,1]$ about the x -axis.

