Math 1210 #2
Limits: An Introduction

Consider this function: $f(x) = \frac{x^2+x-12}{x-3}$

What happens at $x = 3$?

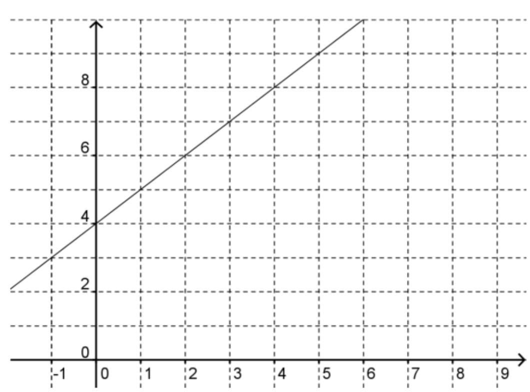
What happens as we approach $x = 3$?

x	$f(x)$
3.25	7.25
3.2	7.2
3.1	7.05
3.05	7.01
3.001	7.001
3	?
2.00	6.99
2.95	6.95
2.9	6.9
2.8	6.8

So we say as x approaches 3, f(x) approaches 7.

Algebraically we compute it this way:

Graphically, it looks like this:



Definition

To say $\lim_{x \to c} f(x) = L$ means that when x is near, but different from c, then $f(x)$ is near L.

EX 1

$$lim_{x \to 2}(3x + 1) =$$

EX 2

$$lim_{x \to 5} \frac{2x^2 - 7x - 15}{x - 5} =$$

EX 3

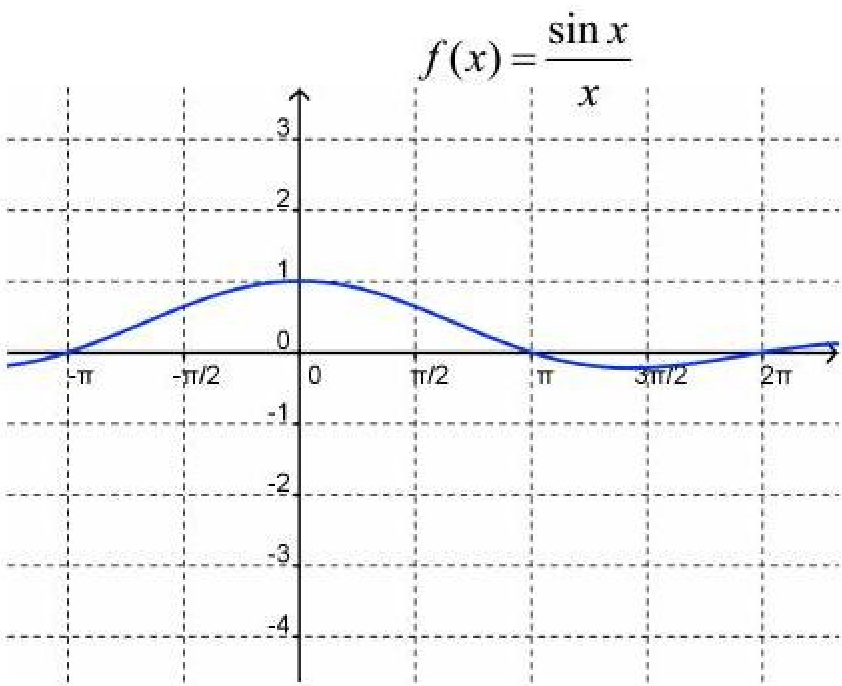
$$lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3} =$$

EX 4

$$lim_{x\to 0} \frac{si\,n\,x}{x} =$$

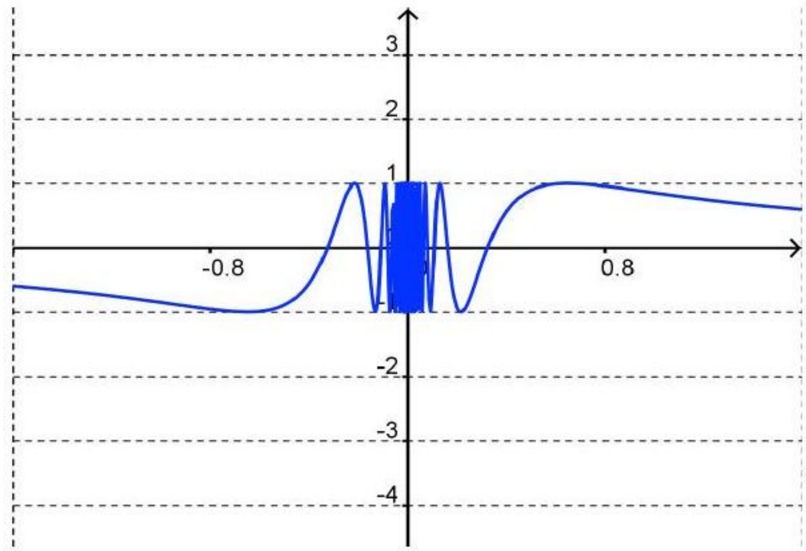
Argument 1		Argument 2
1.0	0.84147	
0.5	0.95885	
0.1	0.99833	
0.01	0.99998	
0	?	
-0.01	0.99998	
-0.1	0.00933	
-0.5	0.05885	
-1.0	0.84147	

Graphically:



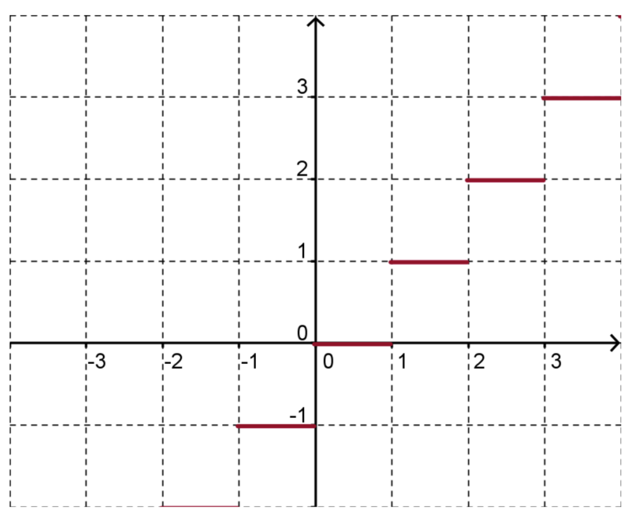
EX 5

$lim_{x \to 0} si\,n\left(\frac{1}{x}\right) =$



EX 6

$lim_{x \to 3} [[x]] =$



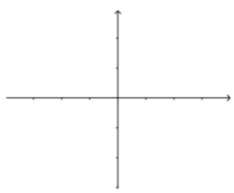
Definition: Right and Left Hand Limits

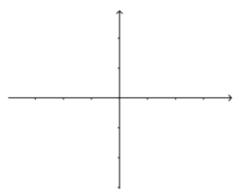
$\lim_{x \to c^+} f(x) = L$ means that when x approaches c from the right side of c, then $f(x)$ is near L.

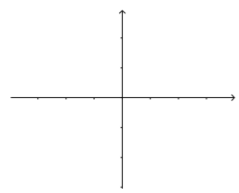
$\lim_{x \to c^-} f(x) = L$ means that when x approaches c from the left side of c, then $f(x)$ is near L.

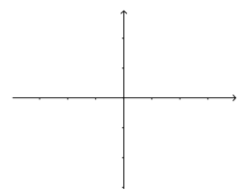
Theorem A

$\lim_{x \to c} f(x) = L$ iff $lim_{x \to c^-} f(x) = L = lim_{x \to c^+} f(x)$

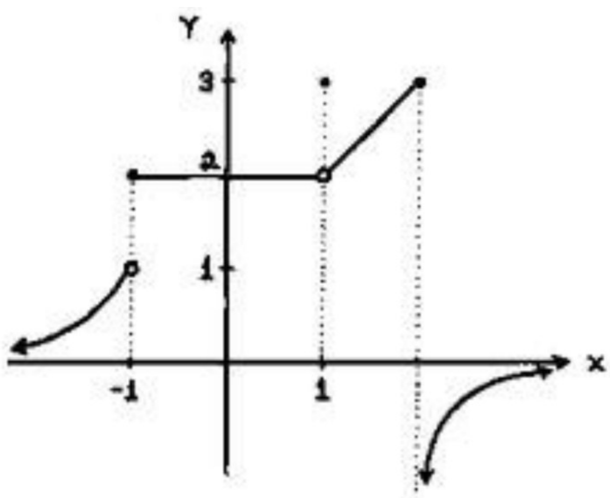








Determine these limits for this function



$\lim_{x \to -1} f(x) =$

$\lim_{x \to -1^-} f(x) =$

$\lim_{x \to -1^+} f(x) =$

$\lim_{x \to 1} f(x) =$

$\lim_{x \to 0} f(x) =$