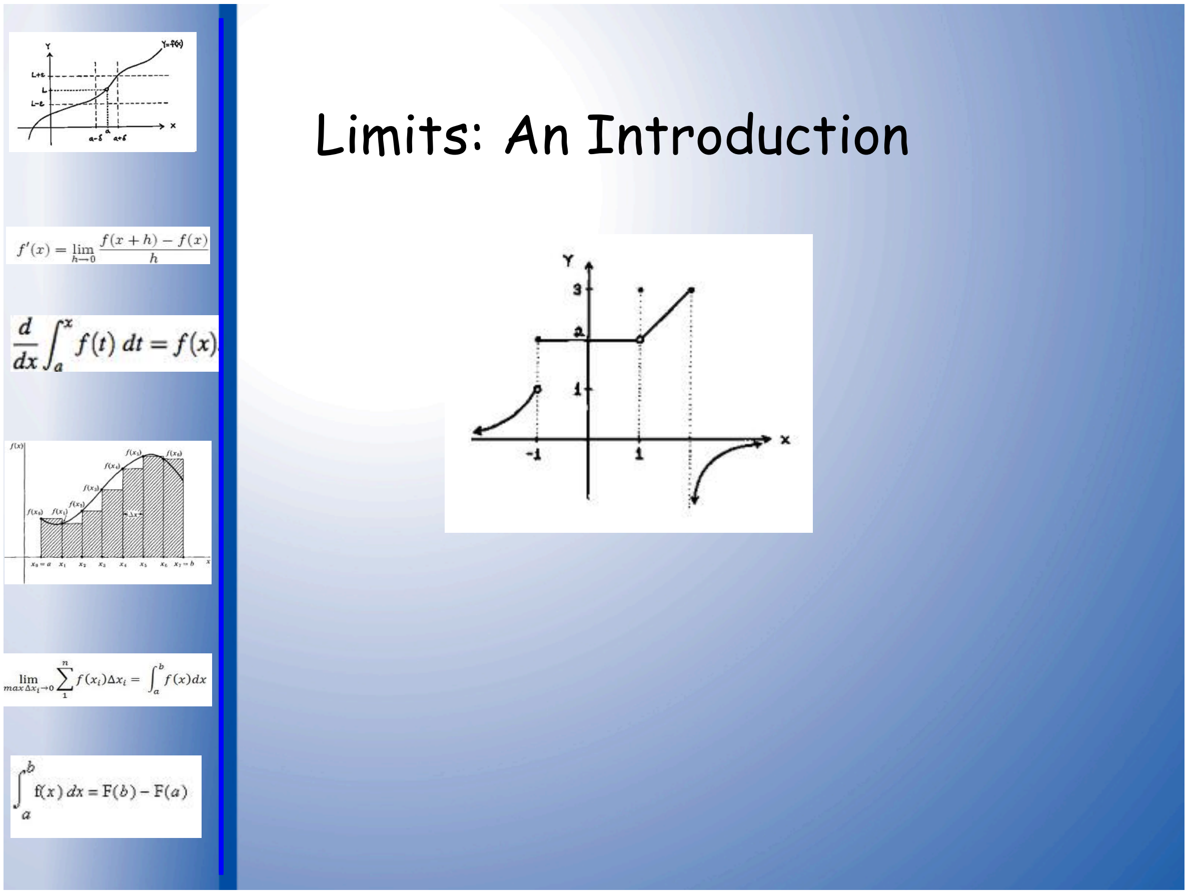
# 2 Introduction to Limits




Consider this function: $f(x) = \dfrac{x^2 + x - 12}{x - 3}$

What happens at x = 3?

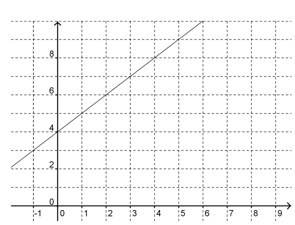
What happens as we approach x = 3?

| x | f(x) |
|---|---|
| 3.25 | 7.25 |
| 3.2 | 7.2 |
| 3.1 | 7.05 |
| 3.05 | 7.01 |
| 3.001 | 7.001 |
| 3 | ? |
| 2.00 | 6.99 |
| 2.95 | 6.95 |
| 2.9 | 6.9 |
| 2.8 | 6.8 |

So we say as x approaches 3, f(x) approaches 7.

Algebraically we compute it this way:

Graphically, it looks like this:



# 2 Introduction to Limits

Definition: To say $\lim_{x \to c} f(x) = L$ means that when *x* is near, but different from *c*,

then *f(x)* is near *L*.

Ex 1 $\lim_{x \to 2}(3x+1) =$

Ex 2 $\lim_{x \to 5} \frac{2x^2 - 7x - 15}{x - 5} =$

Ex 3 $\lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3} =$

Ex 4 $\lim_{x \to 0} \frac{\sin x}{x} =$

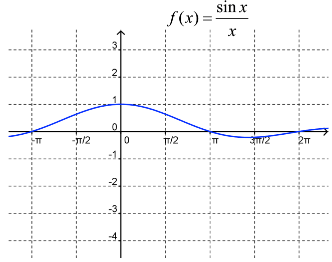
<u>Argument 1</u>

<u>Argument 2</u>

| | |
|---|---|
| 1.0 | 0.84147 |
| 0.5 | 0.95885 |
| 0.1 | 0.99833 |
| 0.01 | 0.99998 |
| 0 | ? |
| -0.01 | 0.99998 |
| -0.1 | 0.00933 |
| -0.5 | 0.05885 |
| -1.0 | 0.84147 |

Graphically:

$f(x) = \frac{\sin x}{x}$



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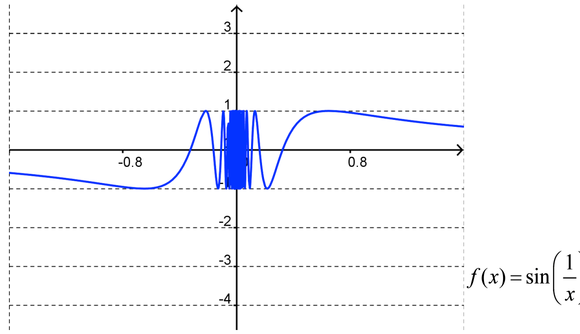
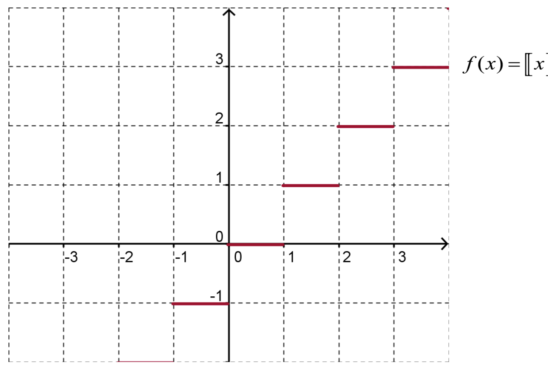
Ex 5 $\lim_{x\to 0} \sin\left(\frac{1}{x}\right) =$




Ex 6 $\lim_{x\to 3} [\![x]\!] =$

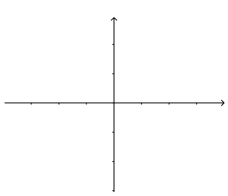
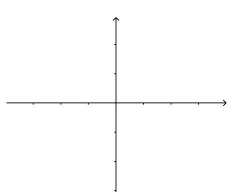
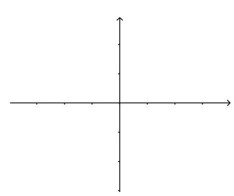
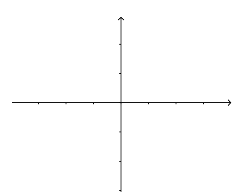



Definition: Right and Left Hand Limits

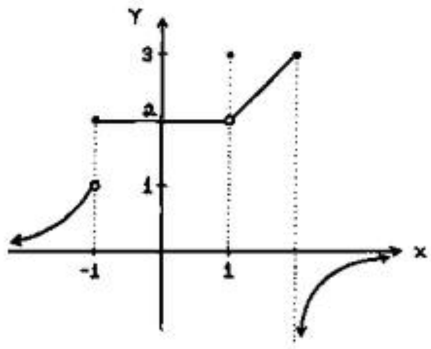
$\lim_{x\to c^+} f(x) = L$ means that when x approaches *c* from the right side of *c*, then *f(x)* is near *L*.

$\lim_{x\to c^-} f(x) = L$ means that when x approaches *c* from the left side of *c*, then *f(x)* is near *L*.

Theorem A $\lim_{x\to c} f(x) = L$ iff $\lim_{x\to c^-} f(x) = L = \lim_{x\to c^+} f(x)$

Determine these limits for this function.




$\lim_{x \to -1} f(x) =$

$\lim_{x \to -1^-} f(x) =$

$\lim_{x \to -1^+} f(x) =$

$\lim_{x \to 1} f(x) =$

$\lim_{x \to 0} f(x) =$