Math 1210 #26
The First Fundamental Theorem of Calculus

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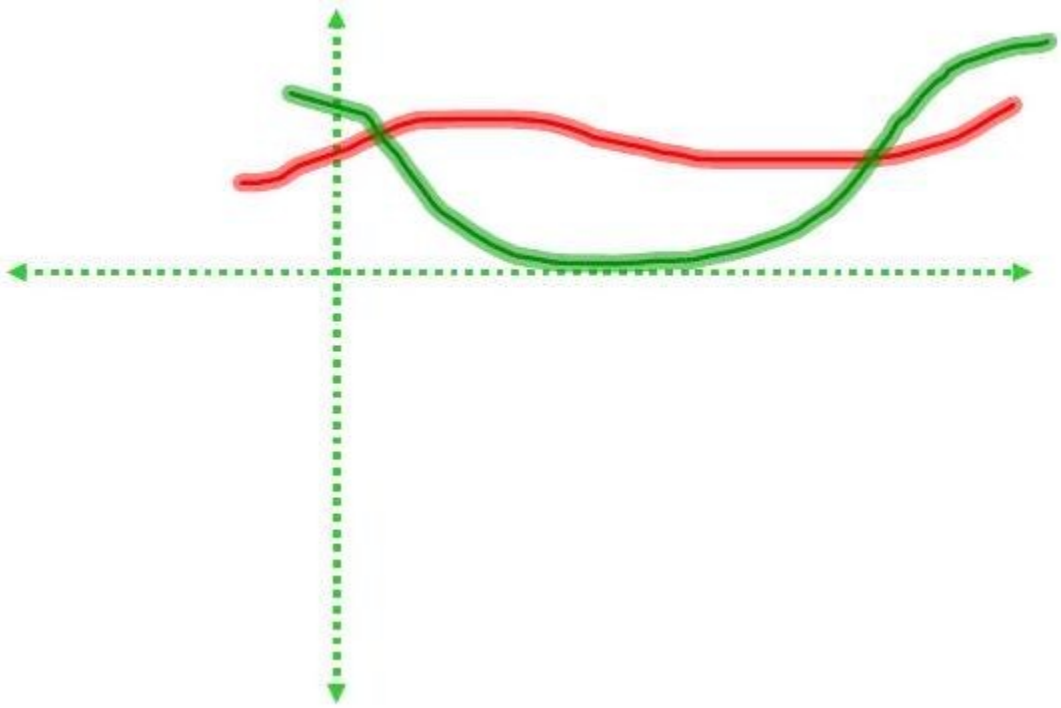
Let f be continuous on [a, b] and let x be a value in (a, b). Then

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

Theorem: Comparison Property

If f and g are integrable on [a, b] and if $f(x) \leq g(x)$ for all x on [a, b], then

$\int_a^b f(x)dx \leq \int_a^b g(x)dx$



Theorem: Boundless Property

If f is integrable on [a, b] and $m \leq f(x) \leq M$ for all a on [a, b],
then $\mathrm{m(b-a)} \leq \int_a^b f(x)dx \leq \mathrm{M(b-a)}$

Theorem: Linearity of the Definite Integral

If f and g are integrable on $[a, b]$ and k is a real number, then

(i) $\int_a^b kf(x)dx = k\int_a^b f(x)dx$
and
(ii) $\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

EX 1

Suppose

$$\int_0^1 f(x)dx = 2 \qquad \int_1^2 f(x)dx = 3$$

$$\int_0^1 g(x)dx = -1 \qquad \int_0^2 g(x)dx = 4$$

Calculate $\int_0^2 (\sqrt{3}f(t) + \sqrt{2}g(t) + \pi)dt$.

EX 2

Find $G'(x)$ for each of these.

2a)

$G(x) = \int_3^x 4tdt$

2b)

$G(x) = \int_1^x (\cos^3(2t)\tan(t))dt \ -\pi/2 < x < \pi/2$

2c)

$G(x) = \int_{-2}^x (xt)dt$

EX 3

Find $\frac{d}{dx}\int_1^{x^2+x} \sqrt{2w + \sin w}dw$

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$