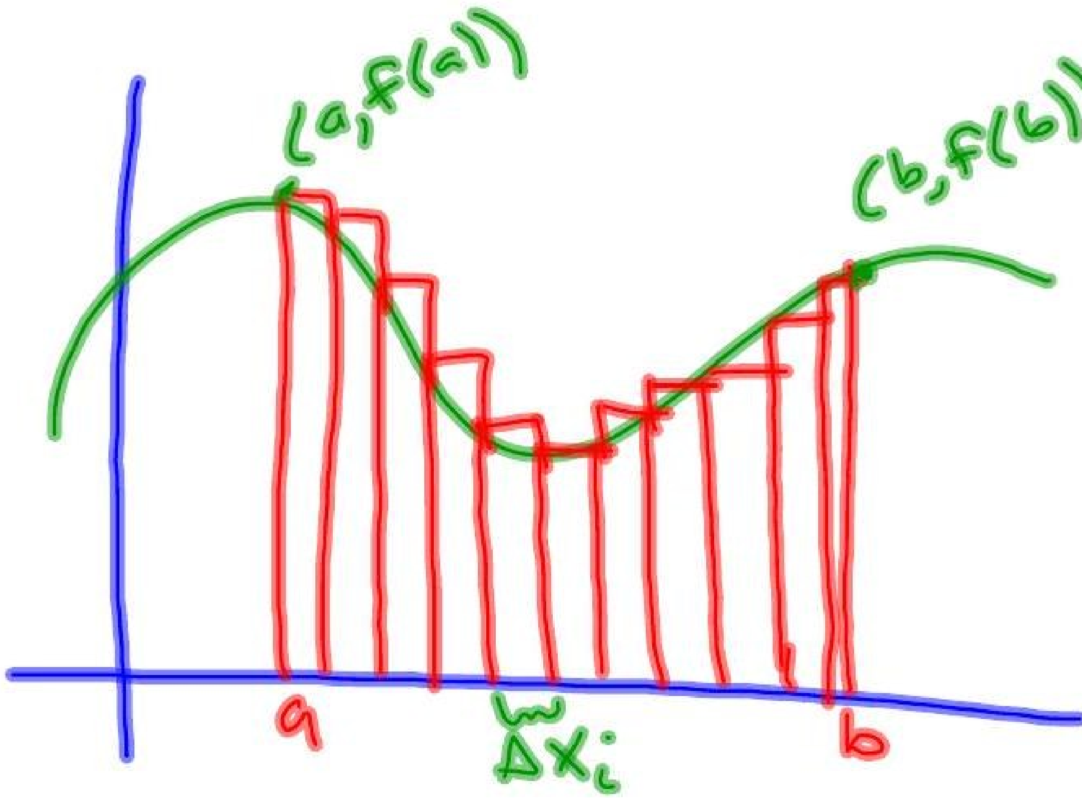
Math 1210 #25
The Definite Integral



Definition of the Definite Integral

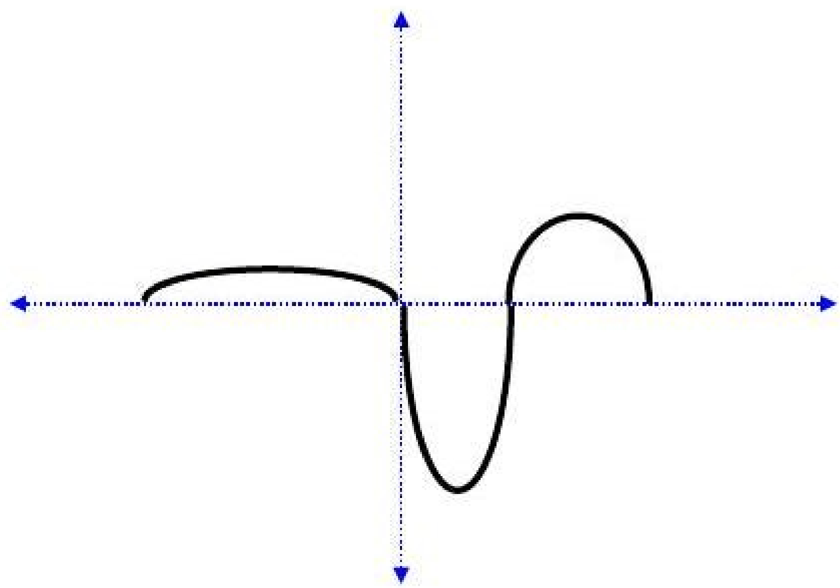
Let f be a function that is defined on [a, b]. If $\lim_{|P|\to 0}\sum_{i=1}^{n} f(\overline{x_i})\Delta x_i$ exists, we say f is integrable on [a, b] and

$$\int_a^b f(x)dx = \lim_{|P|\to 0}\sum_{i=1}^{n} f(\overline{x_i})\Delta x_i$$

$$\int_a^b f(x)dx = A_{up} - A_{down}$$

$$\int_a^a f(x)dx = 0$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$



Integrability Theorem

If f is bounded on $[a, b]$ and continuous there except for a finite number of discontinuities, then f is integrable on $[a, b]$. So, if f is continuous on $[a, b]$ it is integrable on $[a, b]$.

Interval Additive Property

If $f(x)$ is integrable, then $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$.

EX 1

Evaluate this definite integral using the definition.

$$\int_{-1}^{2} (2x - 3)dx$$

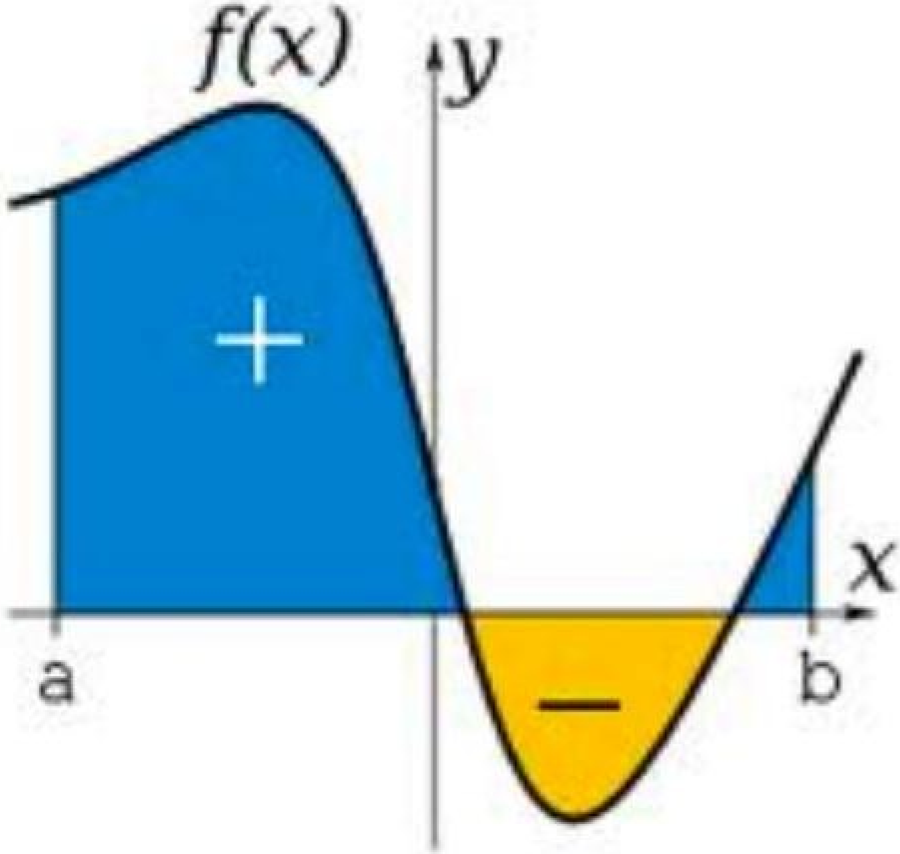
EX 2

Evaluate this definite integral using the definition.

$$\int_0^2 (3x^2 + 2)dx$$

EX 3

Find the area of the region under the curve of $f(x) = -x^2 + 1$ on the interval $[-1,1]$. (To do this, divide the interval $[-1,1]$ into n equal subintervals, calculate the area of the circumscribed or inscribed rectangles and take the limit as $n \to \infty$.)


f(x)
y
+
x
a
−
b