Math 1210 #17
Monotonicity and Concavity

Definition

Let f be defined on an interval I, (open, closed or neither), we say that:

1. f is increasing on I if for every x_1, x_2 in I $x_1 < x_2$ implies $f(x_1) < f(x_2)$.
2. f is decreasing on I if for every x_1, x_2 in I $x_1 < x_2$ implies $f(x_1) > f(x_2)$.
3. f is strictly monotonic on I if it is either increasing or decreasing on I.

Monotonicity Theorem

Let f be continuous on the interval, I and differentiable everywhere inside I.

1. if $f'(x) > 0$ for all x on the interval, then f is increasing on that interval.
2. if $f'(x) < 0$ for all x on the interval, then f is decreasing on that interval.

EX 1

For each function, determine where f is increasing and decreasing.

1a)

$f(x) = x^3 + 3x^2 - 12$

1b)

$$f(x) = \frac{x - 1}{x^2}$$

EX 2

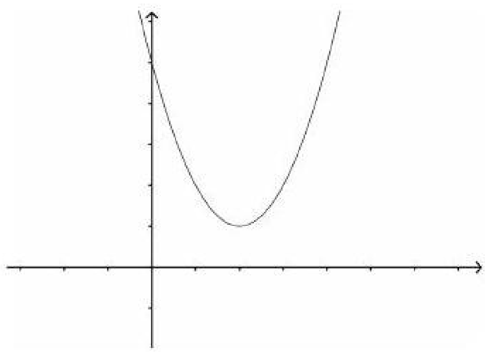
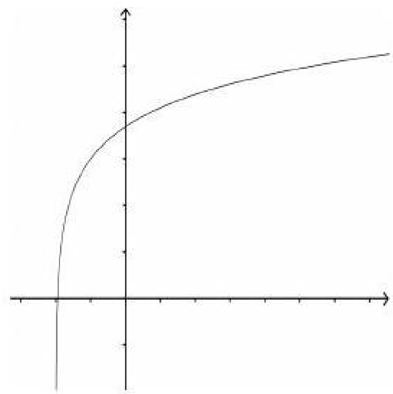
Where is $f(x) = \cos^2 x$ increasing and decreasing on the interval $[0,2\pi]$?

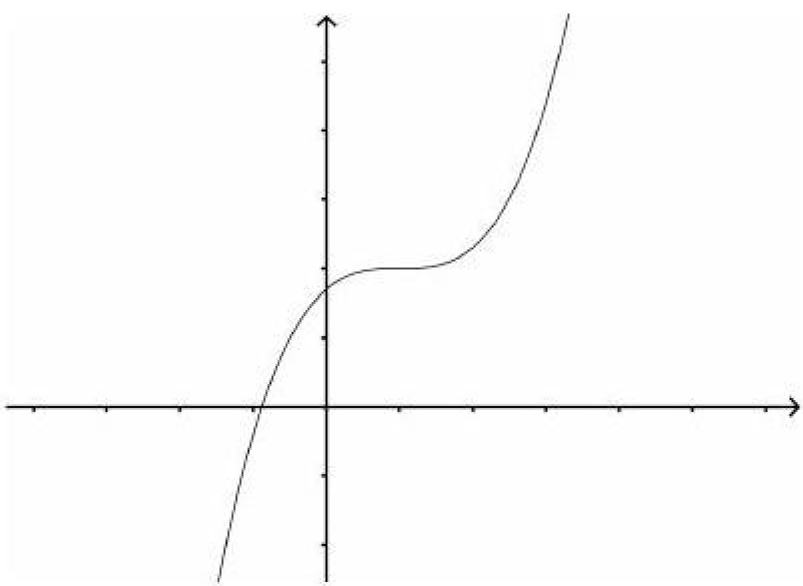
Definition

Let f be differentiable on an open interval, I.

f is concave up on I if $f'(x)$ is increasing on I, and

f is concave down on I if $f'(x)$ is decreasing on I.



Concavity Theorem

Let f be twice differentiable on an open interval, I.
If $f''(x) > 0$ for all x on the interval, then f is concave up on the interval.
If $f''(x) < 0$ for all x on the interval, then f is concave down on the interval.

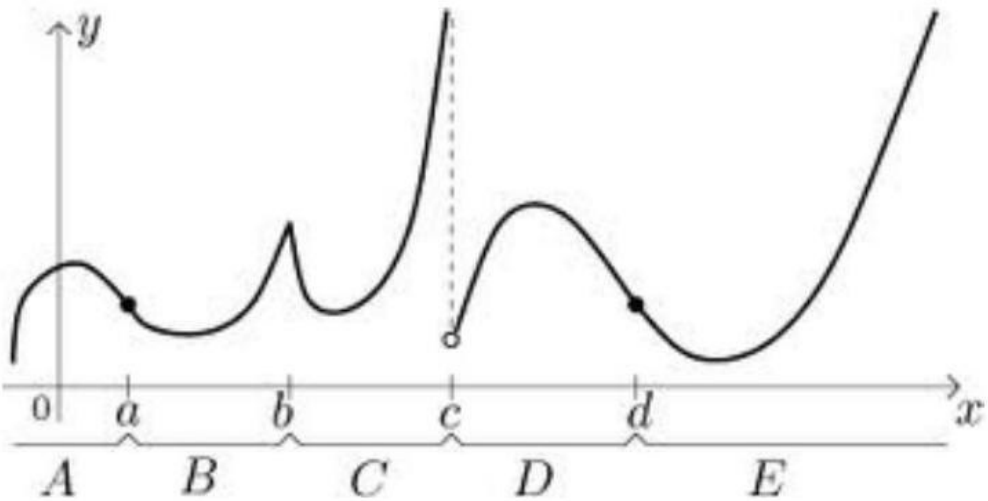
EX 3

Determine where this function is increasing, decreasing, concave up and concave down.

$$f(x) = 4x^3 - 3x^2 - 6x + 12$$

Inflection Point

Let f be continuous at c. We call ($c, f(c)$) an inflection point of f if f is concave up on one side of c and concave down on the other side of c.



Inflection points will occur at x-values for which $f''(x) = 0$ or $f''(x)$ is undefined.

EX 4

For this function, determine where it is increasing and decreasing, where it is concave up and down, find all max/min and inflection points. Use this information to sketch the graph.

$$f(x) = 8x^{1/3} - x^{4/3}$$