Math 1210 #16
Maxima and Minima

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Definition

Let S, the domain of f, contain the point c.
Then

i) $f(c)$ is a maximum value of f on S if $f(c) \geq f(x)$ for all x in S.

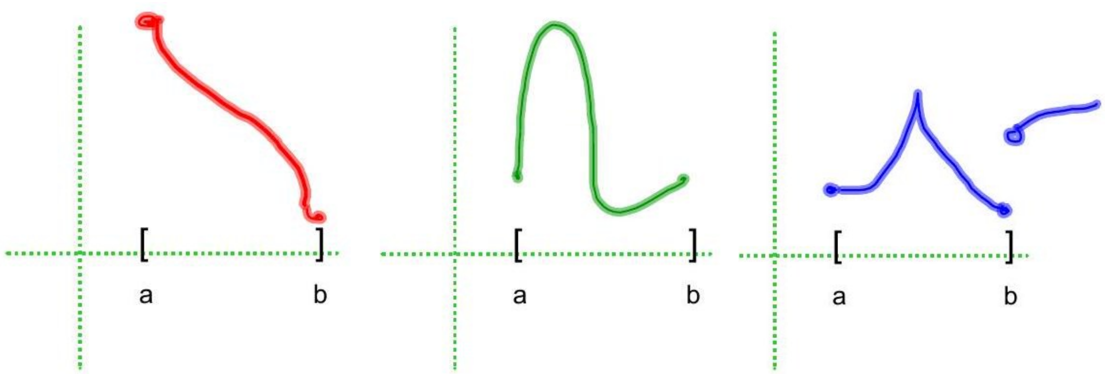
ii) $f(c)$ is a minimum value of f on S if $f(c) \leq f(x)$ for all x in S.

iii) $f(c)$ is an extreme value of f on S if it is the maximum or a minimum value.

iv) the function we want to maximize or minimize is called the objective function.

Maximum - Minimum Existence Theorem

If f is <u>continuous</u> on a closed interval $[a, b]$, then f attains both a maximum and minimum value on that interval.



These can occur in one of three ways:

1. endpoints of the closed interval.
2. stationary points where $f'(x) = 0$.
3. singular points where $f'(x)$ does not exist.

Critical Point Theorem

Let f be defined on a closed interval, I containing the point c. If $f(c)$ is an extreme value, then c is called a critical value.
$(c, f(c))$ is either

1. an endpoint of I or
2. a stationary point of f, i.e., $f'(c) = 0$ or
3. a singular point of f, i.e., $f'(c) DNE$.

EX 1

Find the minimum and maximum values of $f(x) = -2x^3 + 3x^2$ on $[-1,3]$.

EX 2

Find the minimum and maximum points for $f(x) = x^{2/5}$ on $[-1,32]$

EX 3

Show that for a rectangle with perimeter of 30 inches, it has maximum area when it is a square.

EX 4

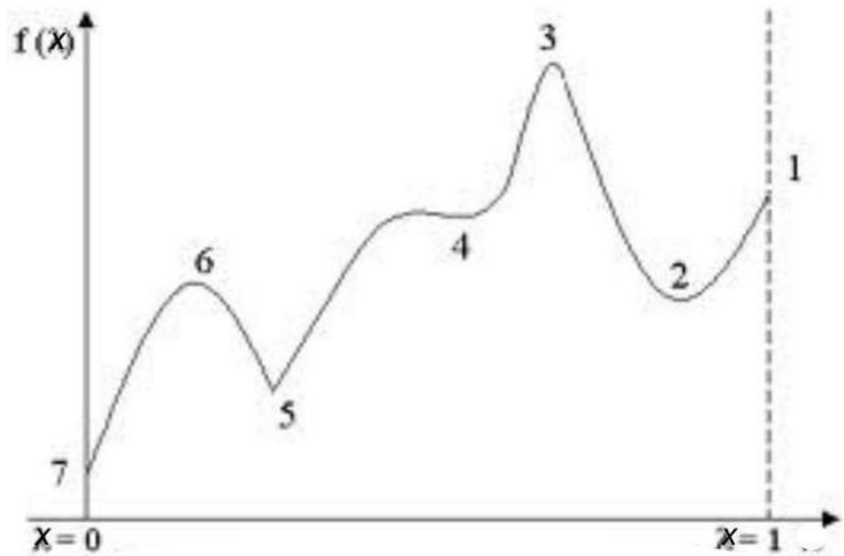
Identify critical points and specify the maximum and minimum values.

$$f(x) = x - 2\sin\ x \ \text{ on } [-2\pi, 2\pi].$$

EX 5

Sketch the graph of a function with all of these characteristics:

1. continuous, but not necessarily differentiable.
2. has domain [0,6]
3. reaches a maximum value of 4 (at $x = 4$)



EX 6

Find all inflection points for $f(x) = 2x^{\frac{1}{3}} - 1$.