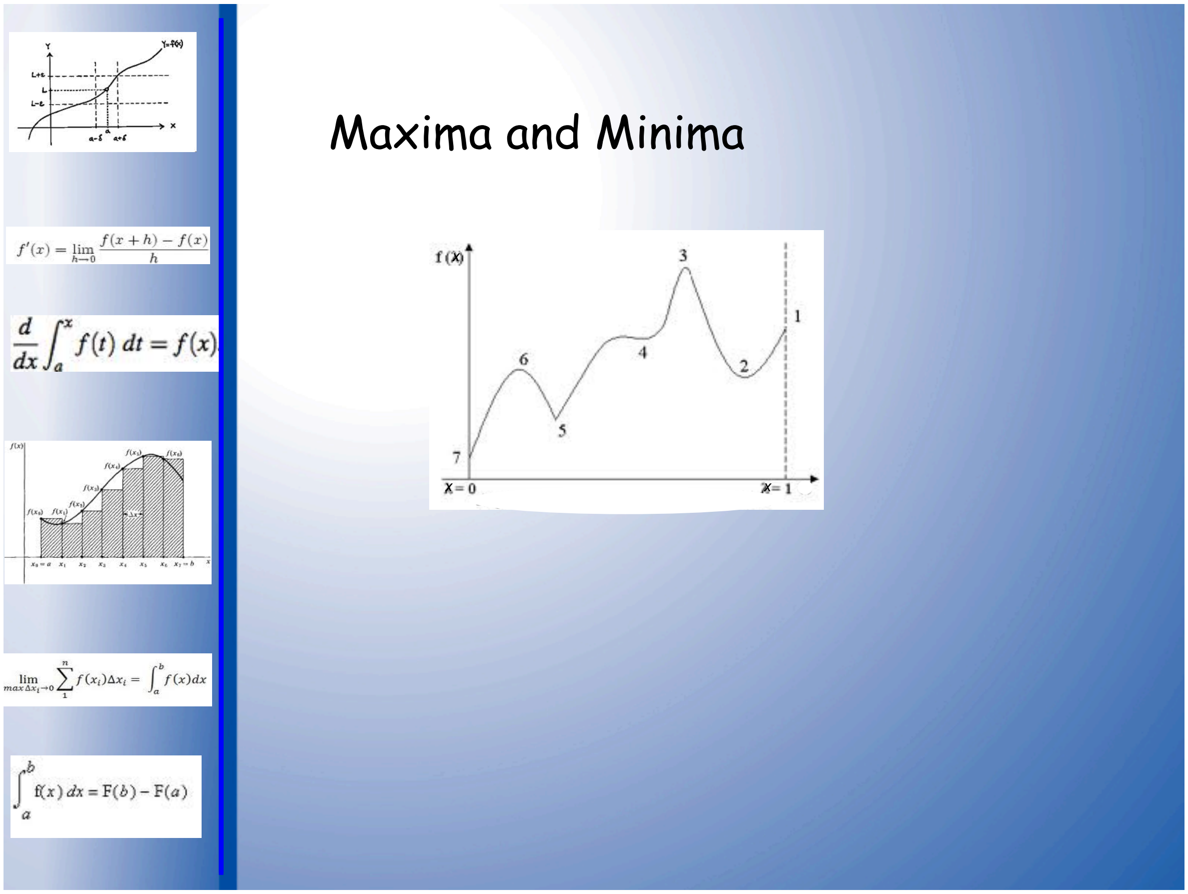


Maxima and Minima

Definition: Let *S*, the domain of *f,* contain the point *c*.

Then

i) *f(c)* is a *maximum* value of *f* on *S* if *f(c)* ≥ *f(x)* for *all x in S.*

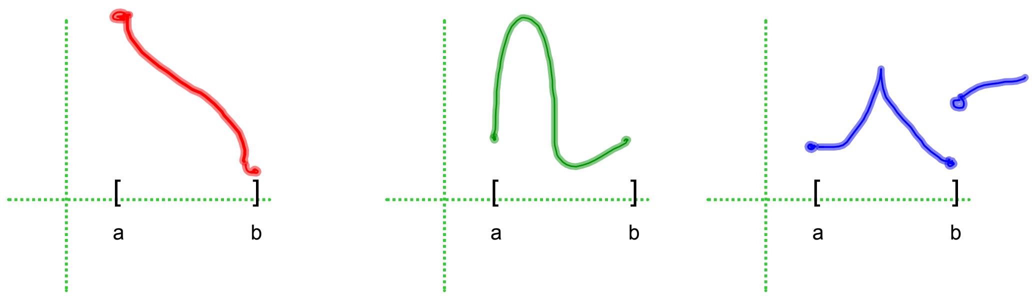
ii) *f(c)* is a *minimum* value of *f* on *S* if *f(c)*≤ *f(x)* for *all x in S.*

iii) *f(c)* is an *extreme* value of *f* on *S* if it is the maximum or a minimum value.

iv) the function we want to maximize or minimize is called the *objective function.*

Maximum - Minimum Existence Theorem

If f is continuous on a closed interval [a,b], then *f* attains both a maximum and minimum value on that interval.



These can occur in one of three ways:

1) endpoints of the closed interval.
2) stationary points where $f'(x) = 0$.
3) singular points where $f'(x)$ does not exist.

Critical Point Theorem

Let *f* be defined on a closed interval, *I* containing the point *c*. If *f(c)* is an extreme value, then *c* is called a critical value.

(c, f(c)) is either

1) an endpoint of I or
2) a stationary point of *f*, i.e., $f'(c)=0$ *or*
3) a singular point of *f*, i.e., *f'(c) DNE.*

Ex 1 Find the minimum and maximum values of $f(x) = -2x^3 + 3x^2$ on [-1,3].

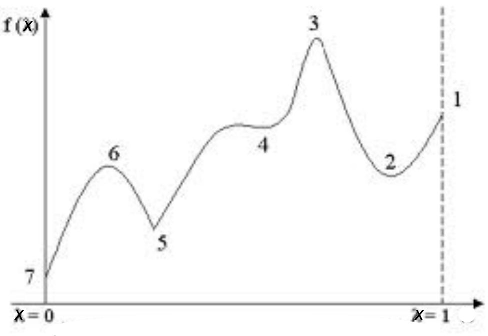
EX 2 Find the minimum and maximum points for $f(x) = x^{2/5}$ on [-1,32]

EX 3 Show that for a rectangle with perimeter of 30 inches, it has maximum area when it is a square.

EX 4 Identify critical points and specify the maximum and minimum values.
$f(x) = x - 2sinx$ on $[-2\pi,\ 2\pi]$.

EX 5 Sketch the graph of a function with all of these characteristics:

1) continuous, but not necessarily differentiable.
2) has domain [0,6]
3) reaches a maximum value of 4 (at x=4)



EX 6 Find all inflection points for $f(x) = 2x^{\frac{1}{3}} - 1$.

16 Maxima Minima