

Math 1210 #15.5

15.5 Differentials

Differentials and Approximations

Differentials and Approximations

We have seen the notation dy/dx and we've never separated the symbols. Now, we'll give meaning to dy and dx as separate entities.

We know $\lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = f'(x_0)$ gives the derivative (slope) of the function $f(x)$ at $x = x_0$.

If Δx is really small, then $\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \approx f'(x_0)$

and $f(x_0 + \Delta x) - f(x) \approx f'(x_0)\Delta x$

Differentials

Let $y = f(x)$ be a differentiable function of x . Δx is an arbitrary increment of x .

$dx = \Delta x$ (dx is called a differential of x .)

Δy is actual change in y as x goes from x to $x + \Delta x$.

i.e. $\Delta y = f(x + \Delta x) - f(x)$

$dy = f'(x)dx$ (dy is called the differential of y .)

EX 1

Find dy .

1a)

$$y = 4x^3 - 2x + 5$$

1b)

$$y = 2\sqrt{x^4 + 6x}$$

1c)

$$y = \cos(x^3 - 5x + 11)$$

1d)

$$y = (x^{10} + \sqrt{\sin(2x)})^2$$

Differentials can be used for approximations.

If $f(x + \Delta x) - f(x) \approx f'(x)\Delta x$,
then $f(x + \Delta x) \approx f(x) + f'(x)\Delta x$.

EX 2

Find a good approximation for $\sqrt{9.2}$ without using a calculator.

EX 3

Use differentials to approximate the increase in the surface area of a soap bubble when its radius increases from 4 inches to 4.1 inches.

EX 4

The height of a cylinder is measured as 12 cm with a possible error of ± 0.1 cm. Evaluate the volume of the cylinder with radius 4 cm and give an estimate for the possible error in this value.

