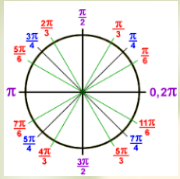
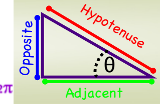
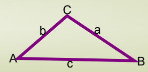
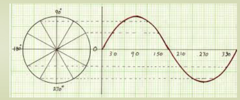








Math 1060 ~ Trigonometry

$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2\sin u\cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

3 The Unit Circle

Learning Objectives

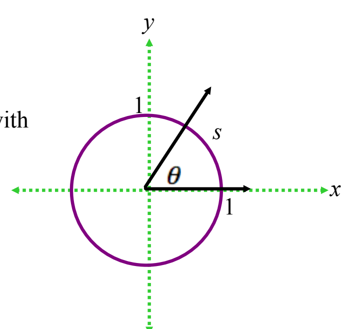
In this section you will:

- **Sketch oriented arcs on the Unit Circle.**
- **Determine the cosine and sine values of an angle from a point on the Unit Circle.**
- **Learn and apply the Pythagorean Identity.**
- **Apply the Reference Angle Theorem.**
- **Learn the cosine and sine values for the common angles whether in degrees or radians.**
- **Learn the signs of the cosine and sine functions in each quadrant.**

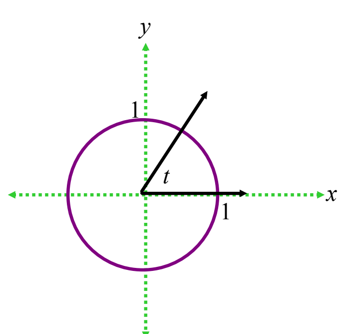
<u>The Unit Circle</u>

Consider the Unit Circle, $x^2 + y^2 = 1$, with angle θ in standard position and the corresponding arc measuring s units in length.

$s = r\theta$

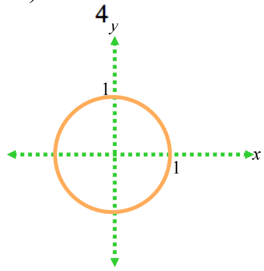


To identify real numbers with oriented angles, we "wrap" the real number line around the Unit Circle and associate to each real number t an oriented arc on the unit circle with initial point (1,0).

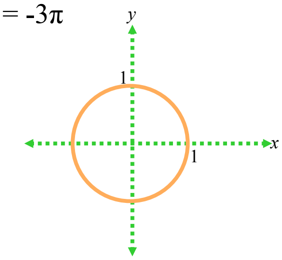


Ex 1: Sketch the oriented arc on the Unit Circle corresponding to each of these real numbers.

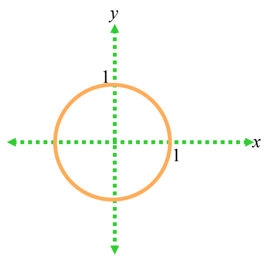
a) $t = \frac{3\pi}{4}$



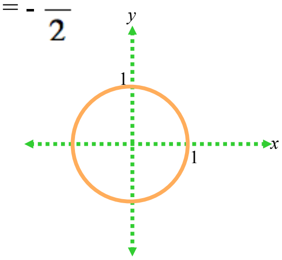
b) $t = -3\pi$



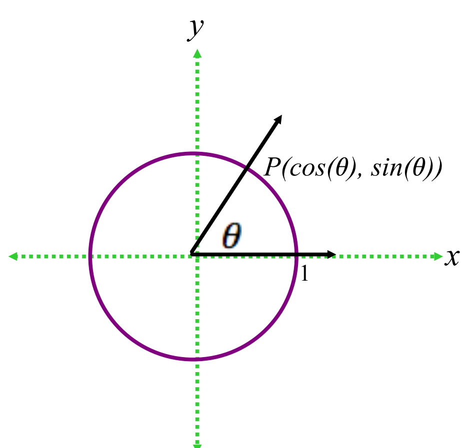
c) $t = 2$



d) $t = -\frac{\pi}{2}$



Determining the cosine and sine functions as points on the Unit Circle.

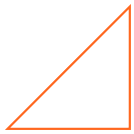


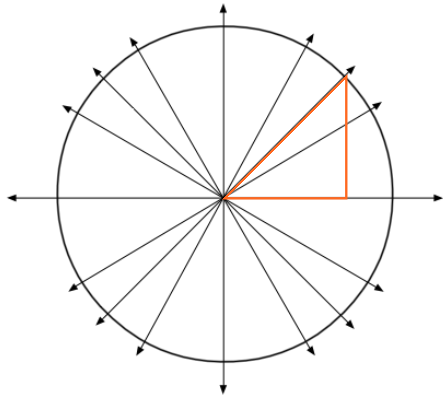
Ex 2:

a) Label the quadrant angles above in radians and degrees and determine the cosine and sine of each.

b) $\cos(-\pi) =$

Question: If the hypotenuse of an isosceles right triangle is 1 unit, how long are each of the legs?



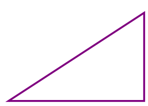


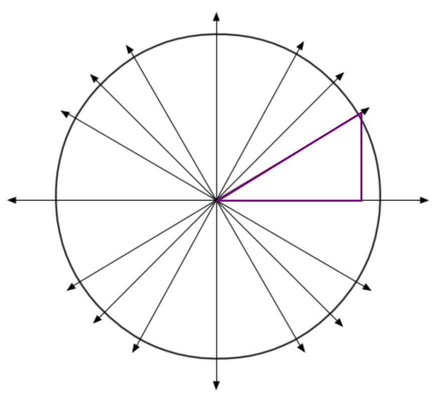
Ex 3:

a) On the figure above, label all the points on the Unit Circle corresponding with angles which are multiples of $\frac{\pi}{4}$.

b) $\sin\frac{5\pi}{4} =$

Question: If the hypotenuse of a right 30º-60º-90º triangle is 1 unit, how long are each of the legs?





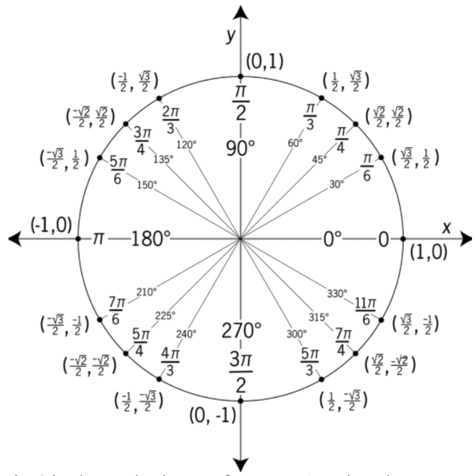
Ex 4:

a) On the figure above, label all the points on the Unit Circle corresponding with angles which are multiples of $\frac{\pi}{6}$.

b) $\cos\frac{5\pi}{6} =$

c) $\sin\frac{2\pi}{3} =$

A complete Unit Circle looks like this.



Given the symmetry of the Unit Circle and The Reference Angle Theorem, you can determine cosine and sine values of these common angles readily.

<u>Reference Angle</u>

A reference angle for a non-terminal angle, θ, is that angle made up of the terminal side of θ and the x-axis.

- It is always positive.
- It is always acute.

<u>Reference Angle Theorem</u>: Suppose α is the reference angle for θ. Then $\cos\theta = \pm\cos\alpha$ and $\sin\theta = \pm\sin\alpha$, where the sign is determined by the quadrant in which the terminal side of θ lies.

Ex 5: For each of the following angles, determine the reference angle and the sine and cosine of each.

Sine and Cosine Values of Common Angles

θ degrees	θ radians	$\cos(\theta)$	$\sin(\theta)$
0°	0	1	0
30°	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
60°	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
90°	$\frac{\pi}{2}$	0	1

a) $\frac{2\pi}{3}$

b) - $\frac{5\pi}{6}$

c) 270º

d) -315º

The Pythagorean Identity

For any angle, θ, $\cos^2\theta + \sin^2\theta = 1$.

Ex 6: Using the given information about θ, find the indicated value.

a) If θ is a second quadrant angle, such that $\sin\theta = \frac{3}{4}$, find $\cos\theta$.

b) If θ is between π and $\frac{3\pi}{2}$ and $\cos\theta = \frac{1}{2}$, find $\sin\theta$.

c) If $\sin\theta = 1$, find $\cos\theta$.