Math 1060 ~ Trigonometry

23 The Dot Product

The dot product of two vectors is a scalar. It can be useful in finding the angle between two vectors.

If $\boldsymbol{v} = \langle v_1, v_2 \rangle$ and $\boldsymbol{w} = \langle w_1, w_2 \rangle$, then $\boldsymbol{v} * \boldsymbol{w} = v_1 w_1 + v_2 w_2$.
Note: $\vec{w} \cdot \vec{w} = w_1 w_1 + w_2 w_2 = w_1^2 + w_2^2 = \|\vec{w}\|^2$ $\|\vec{w}\|^2 = \vec{w} \cdot \vec{w}$

EX 1

Find the dot product of these pairs of vectors.

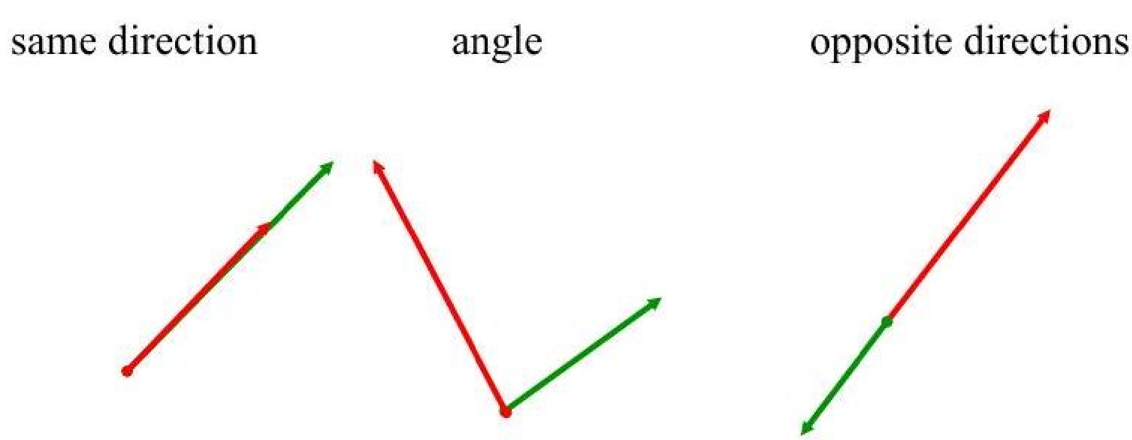
1a)

$\boldsymbol{v} = \langle 3,4 \rangle$ and $\boldsymbol{w} = \langle -2,5 \rangle$.

1b)

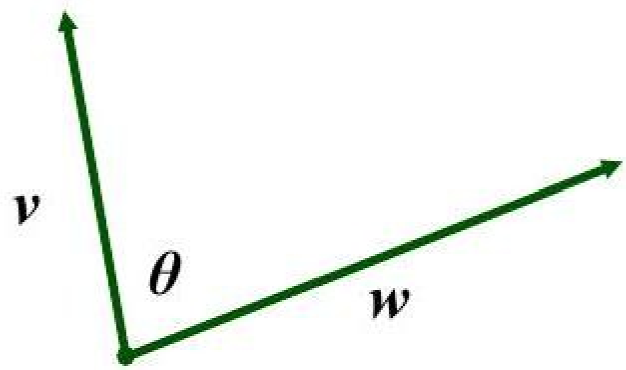
$\boldsymbol{v} = \langle -3,2 \rangle$ and $\boldsymbol{w} = \langle -4,-6 \rangle$

Geometric Interpretation of the Dot Product



Note: there are move dot product properties
listed in the book to refer to.

We will use the Law of cosines to prove that $\boldsymbol{v} \cdot \boldsymbol{w} = \|\boldsymbol{v}\|\|\boldsymbol{w}\|\cos\ \boldsymbol{\theta}, 0 < \theta < \pi$.



$v \cdot w = \|v\|\|w\|\cos\ \theta$

EX 2

Determine the angle between these pairs of vectors.

2a)

$\boldsymbol{v} = \langle 3,4 \rangle$ and $\boldsymbol{w} = \langle -2,5 \rangle$.

2b)

$\boldsymbol{v} = \langle -3,2 \rangle$ and $\boldsymbol{w} = \langle -4,-6 \rangle$

Orthogonal vectors: If two vectors are perpendicular to each other they are said to be orthogonal. What would the cosine of the angle between two orthogonal vectors be?

EX 3

Determine whether these pairs are vectors are orthogonal or not.

3a)

$\langle 3, -2\rangle$ and $\langle 1,4\rangle$

3b)

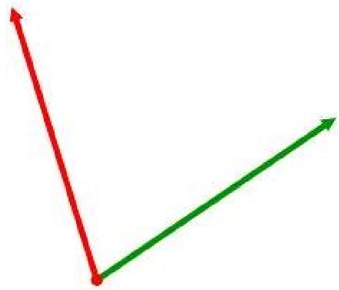
$\langle 4, -6\rangle$ and $\langle -3, -2\rangle$

3c)

$\langle 2, -1\rangle$ and $\langle -4,2\rangle$

Orthogonal Projection

If $\boldsymbol{v}$ and $\boldsymbol{w}$ are nonzero vectors, then the orthogonal projection of $\boldsymbol{v}$ onto $\boldsymbol{w}$, denoted by $\text{proj}_w(\boldsymbol{v})$ is given by



$$proj_w(\vec{v}) = \left(\frac{\vec{v}\cdot\vec{w}}{\|\vec{w}\|^2}\right)\vec{w} = \left[\vec{v}\cdot\left(\frac{\vec{w}}{\|\vec{w}\|}\right)\right]\left(\frac{\vec{w}}{\|\vec{w}\|}\right) = (\vec{v}\cdot\hat{w})\hat{w}$$

EX 4

For $\boldsymbol{v} = \langle -6, -5\rangle$ and $\boldsymbol{w} = \langle 10, -8\rangle$, find $\text{proj}_w(\boldsymbol{v})$.

In physics, you will discover how this concept relates to problems about work.