Math 1060 ~ Trigonometry

19 Trigonometric Representation of Complex Numbers

Review of Complex Numbers

What is i?

The rectangular form of a complex number is $z = a + bi$, where a is the real part and b is the imaginary part. This is represented by $\mathrm{Re}(z) = a$ and $\mathrm{Im}(z) = b$.

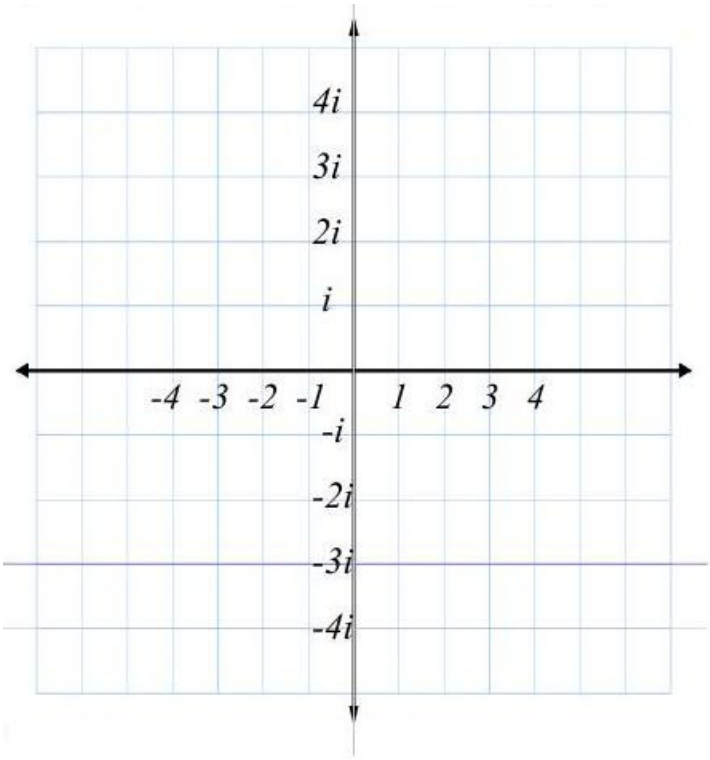
This exercise should serve as a review of complex numbers as learned in a previous course.

EX 1

Let $z_1 = 2 - 2i$ and $z_2 = -3 + 4i$.

1a)

Sketch z_1 and z_2 in the complex plane



1b)

$z_1 + z_2 =$

1c)

$z_1 \times z_2 =$

1d)

$\overline{z_1} =$

1e)

$|z_1| =$

1f)

$(z_1)^2 =$

You may be asking what is the square root of i?

Trigonometric Form (Polar Form) of a Complex Number

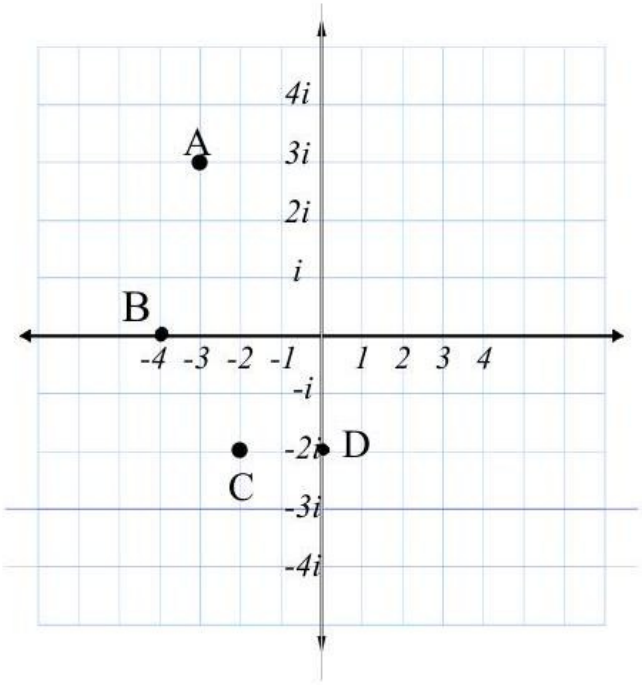
$z = a + bi$ becomes $z = r(\cos\ \theta + i\sin\ \theta) = r\ \text{cis}\ \theta$.

- $r = |z|$ and is called the <u>modulus</u> of z.
- θ is called the <u>argument</u> of z, and $\tan\ \theta = \frac{b}{a}$.
  θ is the angle when sketched in standard position, on the interval $[0,2\pi)$.
  $\tan^{-1}\left|\frac{b}{a}\right|$ will give you the reference angle.
  It is up to you to name the argument in the correct quadrant.

Note that the argument and the modulus are both positive.

EX 2

State the coordinates of these points in rectangular form $(a + bi)$ and in polar form (r cis θ) using radians.



EX 3

Put these in trigonometric (polar) form, $r(\cos\ \theta + i\sin\ \theta)$.

3a)

$z_l = 2\sqrt{3} - 2i$ (radians)

3b)

$z_2 = -3 + 4i$ (degrees)

EX 4

Write these in rectangular form, $(a + bi)$.

4a)

$z_1 = 3\left(\cos\ \frac{5\pi}{3} + i\sin\ \frac{5\pi}{3}\right)$

4b)

$z_2 = 20(\cos\ 210° + i\sin\ 210°)$