Math 1060 ~ Trigonometry

18 Graphing Polar Equations

What do these equations represent?

$\theta = \beta$

$r\cos\ \theta = a$

$r\sin\ \theta = b$

What about these?

$r = 2a\cos\ \theta$

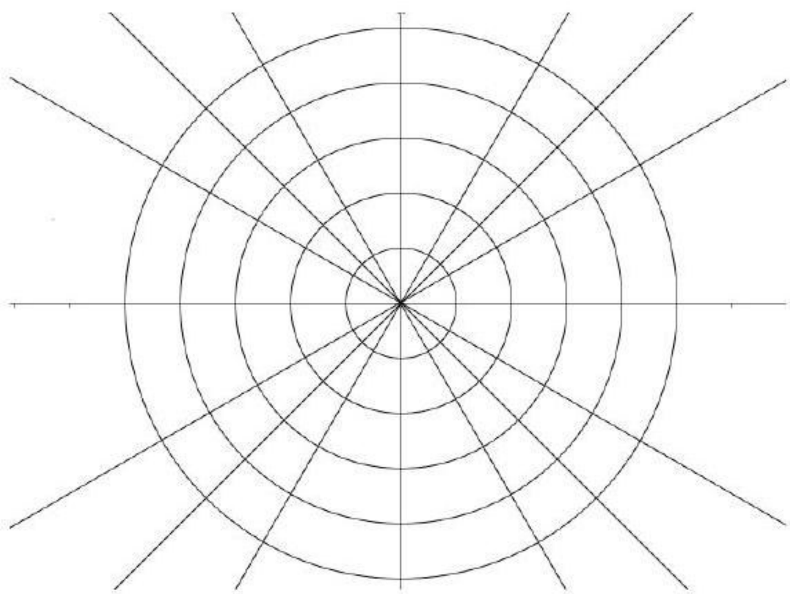
$\mathrm{r} = 2b\sin\theta$

$\mathrm{r} = 2a\cos\theta + 2b\sin\theta$

EX 1

$r = 4\cos\ \theta$

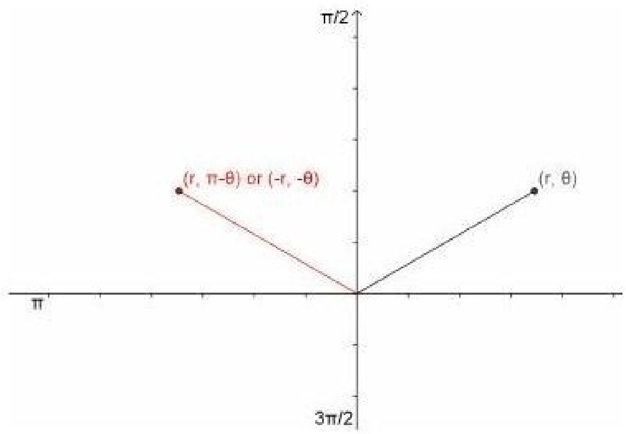
θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
r											



Symmetry

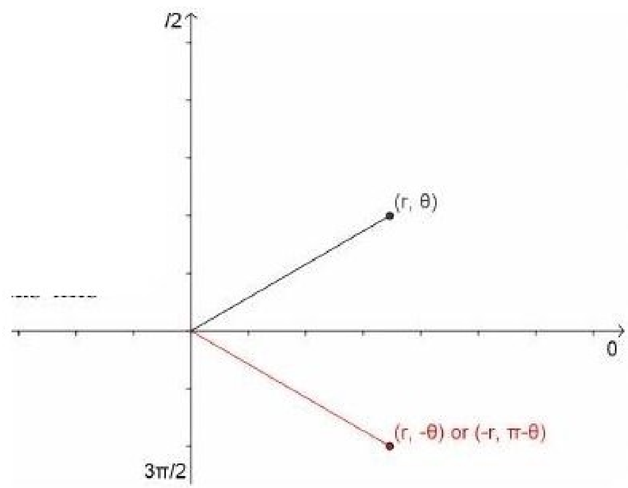
Symmetry with respect to the line θ = π/2

Replace (r, θ) with (r, π-θ) or (-r, -θ): If an equivalent equation results, the graph has this type of symmetry.



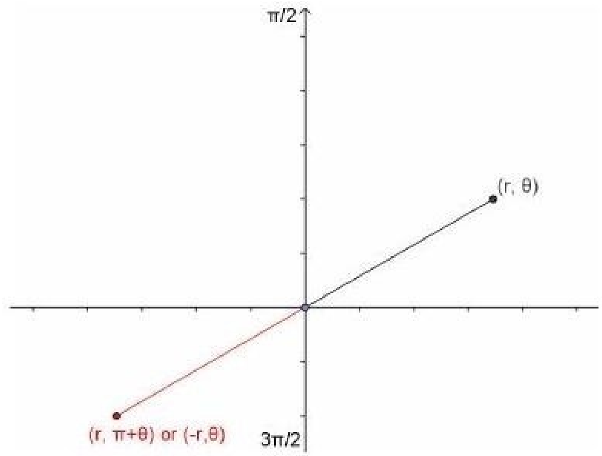
Symmetry with respect to the polar axis (θ = 0):

Replace (r, θ) with (r, -θ) or (-r, π-θ): If an equivalent equation results, the graph has this type of symmetry.



Symmetry with respect to the pole

Replace (r, θ) with (-r, θ) or (r, π+θ): If an equivalent equation results, the graph has this type of symmetry.



If a polar equation passes a symmetry test, then its graph definitely exhibits that symmetry. However, if a polar equation fails a symmetry test, then its graph may or may not have that kind of symmetry.

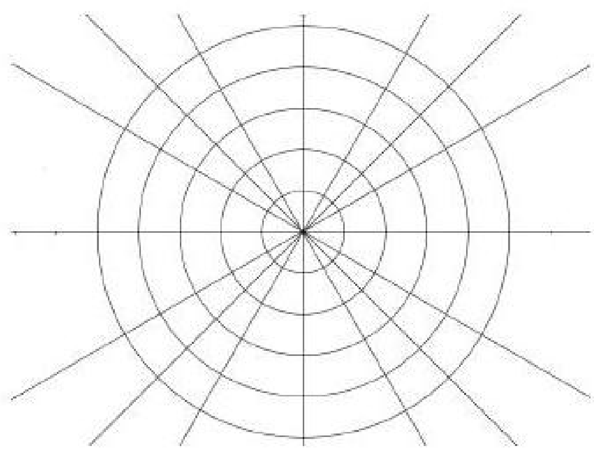
Zeros and maximum *r*-values

Other helpful tools in graphing polar equations are knowing the values for θ for which |r| is maximum and those for which r = 0.

EX 2

Graph $r = \frac{1}{2} + cos\theta$

Symmetry:



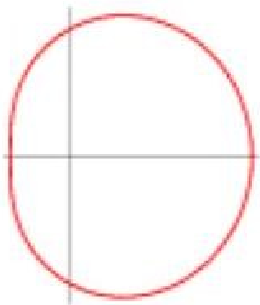
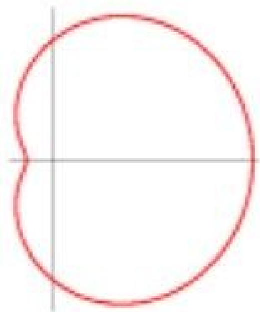
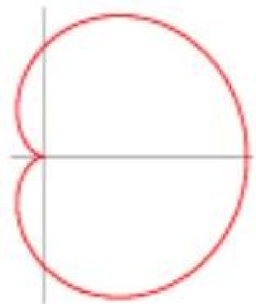
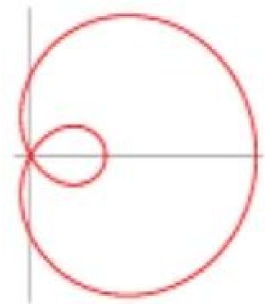
|r| maximum:

Zero of r:

Limaçons

$r = a \pm b\cos\theta$

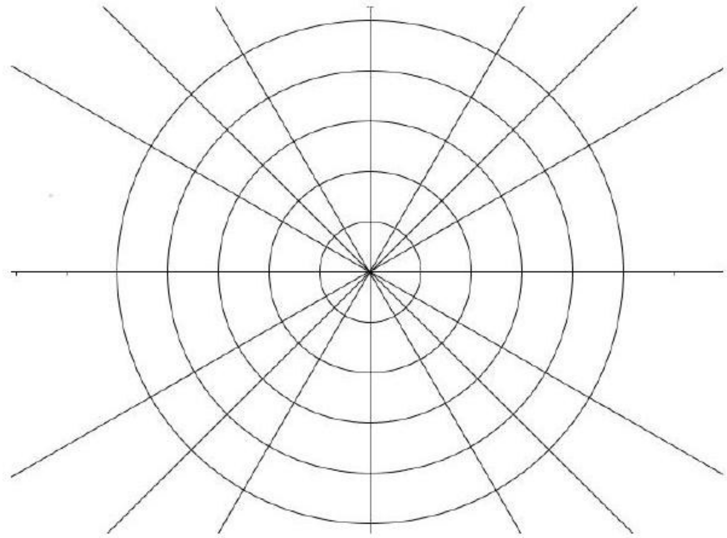
$r = a \pm b\sin\theta \qquad a > 0, b > 0$

			
$\frac{a}{b} \geq 2$	$1 < \frac{a}{b} < 2$	$\frac{a}{b} = 1$	$\frac{a}{b} < 1$
Convex limaçon	Dimpled limaçon	Cardioid - always passes through pole	Limaçon with inner loop

EX 3

Graph r = 3 sin 2θ

Symmetry:



|r| maximum:

Zero of r:

θ	0	$\pi/8$	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
r						

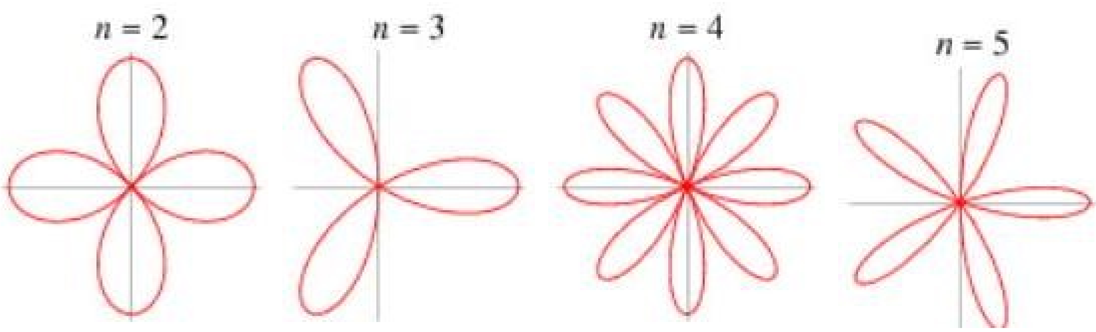
Roses

$$r = a\sin(n\theta),$$

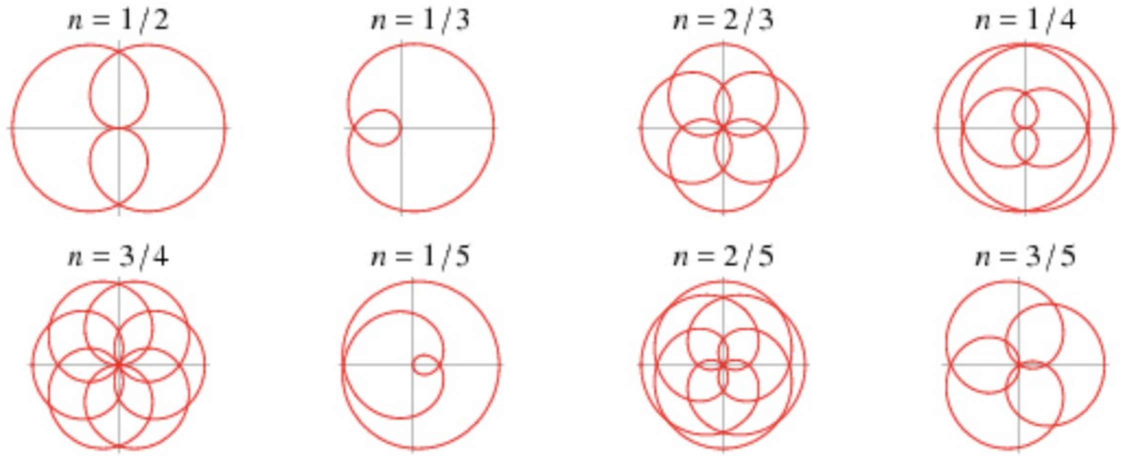
or

$$r = a\cos(n\theta)$$

If n is odd, the rose is n-petalled. If n is even, the rose is $2n$-petalled.



No reason to limit ourselves to n integer:



Or even irrational: