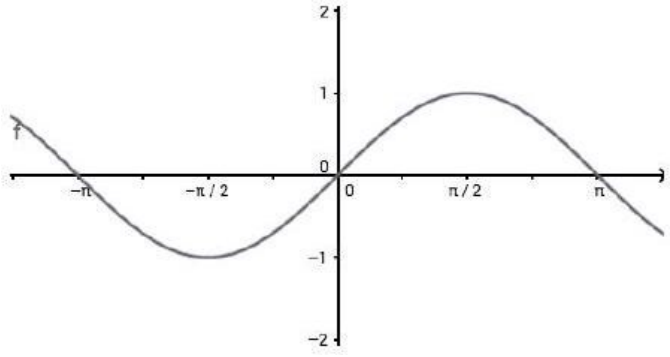
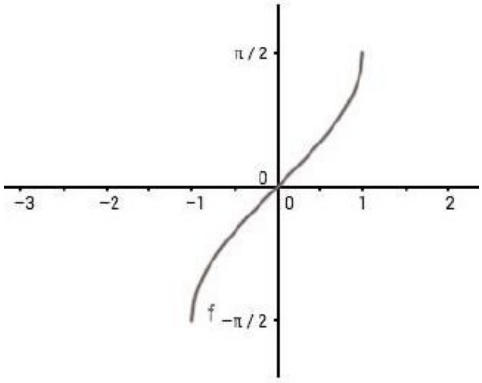
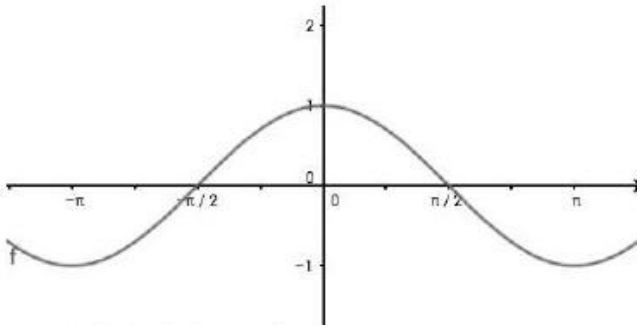
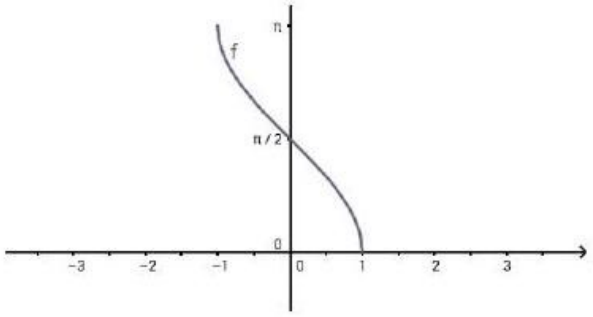
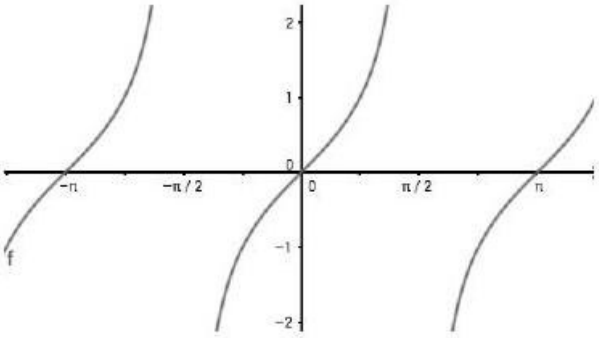
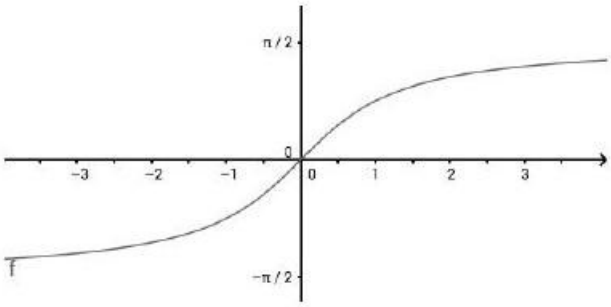
Math 1060 ~ Trigonometry

12 Inverse Trigonometric Functions

To find the inverse of the trigonometric functions, our first problem is that they are not one-to-one.

$y = \sin(x)$	$y = \sin^{-1}(x) = \arcsin(x)$
	
restricted domain:	domain:
range:	range:
symmetry:	symmetry:
does $\sin(\sin^{-1}(x)) = x$?	does $\sin^{-1}(\sin(x)) = x$?

$y = \cos(x)$	$y = \cos^{-1}(x) = \arccos(x)$
	
restricted domain:	domain:
range:	range:
symmetry:	symmetry:
does $cos(\cos^{-1}(x)) = x$?	does $\cos^{-1}(\cos(x)) = x$?

$y = \tan(x)$	$y = \tan^{-1}(x) = \arctan(x)$
	
restricted domain:	domain:
range:	range:
symmetry:	symmetry:
does $\tan(\tan^{-1}(x)) = x$?	does $\tan^{-1}(\tan(x)) = x$?

When working these problems, it is easier if you think of the Unit Circle rather than the Cartesian graph.

$\sin^{-1}(x)$ answers will be in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1}(x)$ answers will be in the interval $[0, \pi]$

$\tan^{-1}(x)$ answers will be in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Note that to compute the $\sec^{-1}(x)$, $\csc^{-1}(x)$ and $\cot^{-1}(x)$ you can turn each into a problem involving the three functions above.

EX 1

Look at a Unit Circle and practice by finding the answers to these:

1a)

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

1b)

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

1c)

$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

1d)

$\sin^{-1}\left(-\frac{1}{2}\right)$

1e)

$\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

1f)

$\tan^{-1}(-1)$

EX 2

Try these without looking at a Unit Circle.

2a)

$\sin^{-1}(-1)$

2b)

$\cos^{-1}(0)$

2c)

$\tan^{-1}(1)$

2d)

$\csc^{-1}(0)$

2e)

$\sec^{-1}(-2)$

2f)

$\cot^{-1}(-1)$

EX 3

Which of these are true? Correct any that are false.

3a)

$$\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right) = \frac{3\pi}{4}$$

3b)

$$\cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right) = \frac{1}{2}$$

3c)

$\tan^{-1}(\tan\pi) = \pi$

EX 4

These will require a bit more thought and perhaps a drawing of a triangle.
Evaluate these.

4a)

$\cos\left(\arctan\left(\frac{2}{3}\right)\right)$

4b)

$\tan\left(\sin^{-1}\left(\frac{3}{4}\right)\right)$

4c)

$\sec\left(\cos^{-1}\left(\frac{3x}{2}\right)\right)$

EX 5

Evaluate these.

5a)

$\sec\left(\arctan\left(-\frac{3}{4}\right)\right)$

5b)

$\cot\left(\sin^{-1}\left(-0.2\right)\right)$

EX 6

Here is another challenge. Evaluate these.

6a)

$\sec\left(\arctan\left(-\frac{3}{4}\right)\right)$

6b)

$\cot\left(\sin^{-1}(-0.2)\right)$

EX 7

A plane flies at an altitude of 6 miles toward a point directly over an observer. Write the angle θ as a function of x , the horizontal distance from the observer to a point on the ground directly below the airplane.