

# Math 1060 ~ Trigonometry

## 11 Multiple Angle Identities

The double angle identities are easy to generate using the identities for the sum of two angles.

$$\sin (2\theta) = \sin (\theta + \theta)$$

$$\cos (2\theta) = \cos (\theta + \theta)$$

$$\tan (2\theta) = \tan (\theta + \theta)$$

Double Angle Identities: For all applicable angles  $\theta$ ,

- $\sin (2\theta) = 2\sin (\theta)\cos (\theta)$
- $\cos (2\theta) = \begin{cases} \cos^2 (\theta) - \sin^2 (\theta) \\ 2\cos^2 (\theta) - 1 \\ 1 - 2\sin^2 (\theta) \end{cases}$
- $\tan (2\theta) = \frac{2\tan (\theta)}{1-\tan^2 (\theta)}$

Why do we need the double angle identities? Do they allow us to compute exact values of any angles?

- Simplify expressions.
- Solve equations with  $2x$ .

### **EX 1**

Solve this equation for values of  $x$  on the interval  $[0, 2\pi)$ .

$$\sin(2x) + \cos(x) = 0$$

Starting with two forms of the double angle identity for the cosine, we can generate half-angle identities for the sine and cosine.

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

**Power Reduction Formulas:** For all angles  $\theta$ ,

- $\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$
- $\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$

$$\sin\left(\frac{\theta}{2}\right)$$

$$\cos\left(\frac{\theta}{2}\right)$$

$$\tan\left(\frac{\theta}{2}\right)$$

**Half Angle Formulas.** For all applicable angles  $\theta$ ,

- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$
- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$
- $\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}$

where the choice of  $\pm$  depends on the quadrant in which the terminal side of  $\frac{\theta}{2}$  lies.

## EX 2

Use these identities to determine exact values.

### 2a)

$$\sin(105^\circ)$$

### 2b)

$$\tan\left(\frac{7\pi}{12}\right)$$

### EX 3

If  $\theta$  is an obtuse angle and  $\sin \theta = \frac{3}{5}$ , find the exact value of these using double/half angle identities.

**3a)**

$$\sin(2\theta)$$

**3b)**

$$\cos\left(\frac{\theta}{2}\right)$$

**3c)**

$$\tan(2\theta)$$

### EX 4

Evaluate  $\cos\left(\frac{7\pi}{12}\right)$  in two ways, using the half-angle identity and using the difference identity.