Math 1060 ~ Trigonometry

10 Verifying Trigonometric Identities

Identities are useful in simplifying expressions and computing values for some of the less-familiar angles on the Unit Circle. In this video, we will familiarize ourselves with some important identities and use them to compute values, simplify expressions and verify other identities.

The Even/Odd Identities: For all applicable angles θ

$\sin(-\theta) = -\sin(\theta)$	$\cos(-\theta) = \cos(\theta)$	$\tan(-\theta) = -\tan(\theta)$
$\csc(-\theta) = -\csc(\theta)$	$\sec(-\theta) = \sec(\theta)$	$\cot(-\theta) = -\cot(\theta)$

EX 1

Simplify these expressions.

1a)

$\sin(-x)\cos(-x)\sec(-x)$

1b)

$-\cot(-x)\tan(x)$

Let's play a little True False game.

EX 2

Identify these as True or False:

2a)

$5(c + d) = 5c + 5d$

2b)

$\frac{a+b}{2} = \frac{a}{2} + \frac{b}{2}$

2c)

$(x + y)^2 = x^2 + y^2$

2d)

$\sqrt{p+q} = \sqrt{p} + \sqrt{q}$

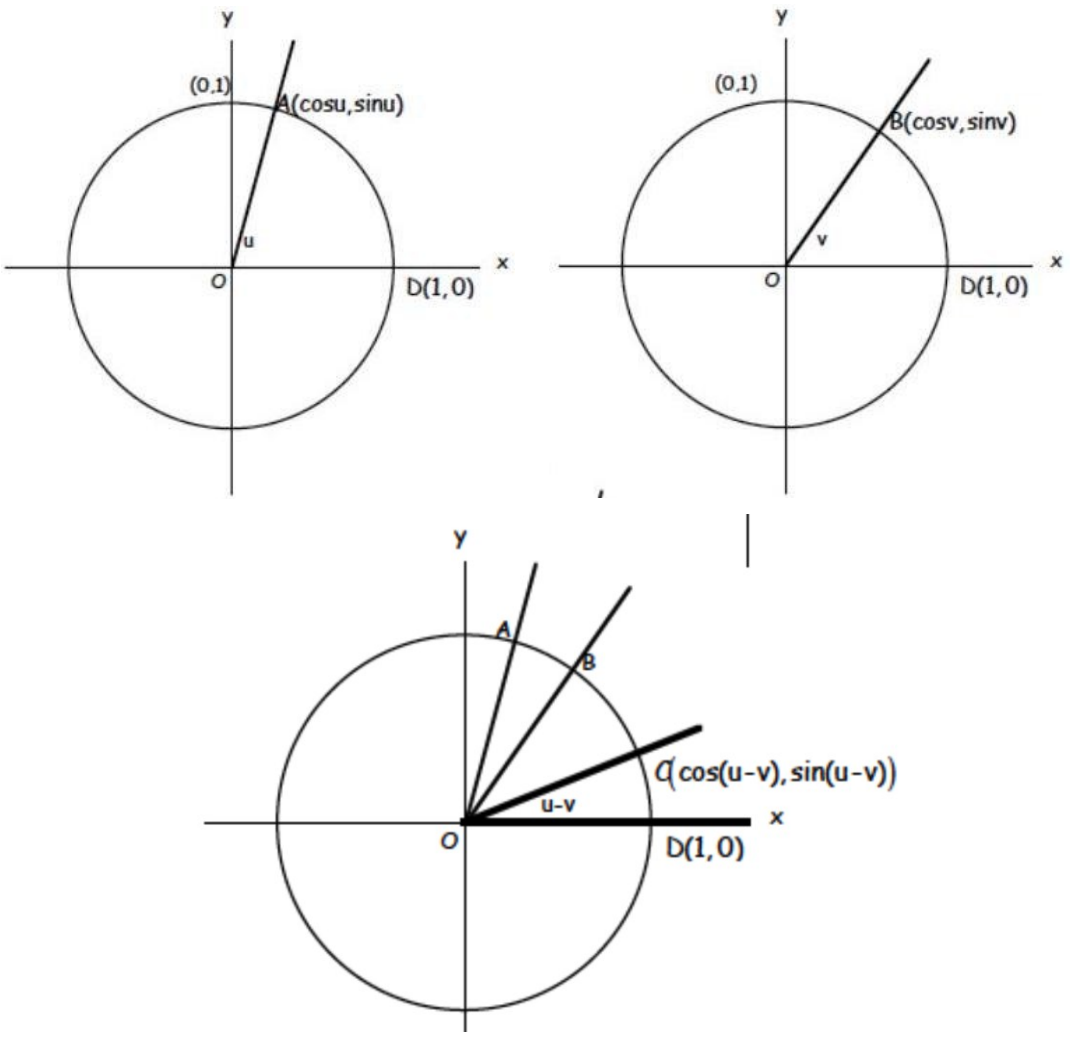
2e)

$\sin(u + v) = \sin u + \sin v$

We can tell by inspection that $\sin 30° + \sin 60° \neq \sin 90°$.

So, we need a set of sum/difference identities for the sine.
Sum and Difference Identities: For all applicable angles α and β,

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
- $\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$



Distance from A to B

$\sqrt{(\cos\ u - \cos\ v)^2 + (\sin\ u - \sin\ v)^2}$

Distance from C to D

$= \sqrt{(\cos\ (u - v) - 1)^2 + (\sin\ (u - v) - 0)^2}$

Sum and Difference Identities for Sine. For all angles α and β,

- $\sin\ (\alpha + \beta) = \sin\ (\alpha)\cos\ (\beta) + \cos\ (\alpha)\sin\ (\beta)$
- $\sin\ (\alpha - \beta) = \sin\ (\alpha)\cos\ (\beta) - \cos\ (\alpha)\sin\ (\beta)$

EX 3

Determine these exact values of $\sin 75°$ in two ways.

3a)

$\sin(75°) = \sin(30° + 45°)$

3b)

$\sin(75°) = \sin(120° - 45°)$

Sum and Difference Identities: For all applicable angles α and β,

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
- $\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$

EX 4

Use the sum/difference identities to find the exact values of these.

4a)

$\cos \frac{5\pi}{12}$

4b)

$\sin\left(-\frac{7\pi}{12}\right)$

EX 5

tan 165°

EX 6

Verify this cofunction identity.

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$